Modeling of Fairness – Distribution of Efficiency Gains in Supply Webs from a Game-theoretic Point of View –

Stephan Zelewski [1], Malte L. Peters [2]

[1] Institute for Production and Industrial Information Management, University of Duisburg-Essen, Campus Essen, Universitätsstrasse 9, D-45141 Essen, Germany
stephan.zelewski@pim.uni-due.de
[2] Institute for Production and Industrial Information Management, University of Duisburg-Essen, Campus Essen, Universitätsstrasse 9, D-45141 Essen, Germany
malte.peters@pim.uni-due.de

Abstract

The addressed *scientific problem* is the purpose to distribute the efficiency gains among partially autonomous actors in a supply web in a manner that the actors accept as fair and thus find it advantageous to cooperate with each other.

There are three reasons why this problem is economically significant. Firstly, it is required to substantiate efficiency gains in an understandable manner. Secondly, supply webs suffer from a conflict potential, since the partially autonomous actors are trying to maximize their own shares of the efficiency gain. Thirdly, the cooperative game theory offers some solution approaches for distribution problems. These are especially the SHAPLEY value and the Nucleolus. These approaches suffer from a serious drawback: The fairness of the distribution results is hard to justify, since the logic of these approaches is difficult to communicate.

The *scientific method* applied is (i) in order to solve the substantiation problem and (ii) reconstructing the distribution problem through a game-theoretic model on basis of the τ-value. The special nature of the τ-value ensures that it seems rational to the actors to cooperate, because the distribution results are obtained by successively restricting the solution space with the help of easily understandable rationality principles and integrity conditions.

The *expected scientific result* is that it is shown how the monetary value of efficiency gains in supply webs can be established precisely. Moreover, the proposed solution method for the distribution problem offers a fair distribution of efficiency gains and ensures that the distribution results can be communicated easily.

Keywords: bullwhip effect, efficiency gains, game theory, supply chain management, supply web, τ-value.

1. Distribution of efficiency gains in supply webs

Recently much research has been done on the management of supply chains and supply webs. The primary aim of this research is to find ways to realize efficiency gains by coordinating the activities of all supply chain participants.

Several effects have to be taken into account as sources of efficiency gains. The probably most prominent effect is the so called bullwhip effect. The bullwhip effect [8; see also 7, 10, 18] describes especially how companies build inventory buffers based on the demand of their customers and that the further the company is from the final customer the greater is the "safety stock" in times of rising demand. Notably the cost of capital invested in inventory buffers in the stocks is causing inefficiency and thus efficiency gains can be realized by avoiding or reducing the bullwhip effect. Above all ZÄPFEL has done a lot of research on supply chain management and especially on the bullwhip effect as well as on the decision behavior of actors in supply chain management [15, 16]. ZÄPFEL and PIEKARZ [17] have developed simulation models in the area of supply chain controlling, which take the decision interdependencies between the activities of the actors in the supply chain into account. Furthermore, ZÄPFEL and WASNER [18] have conceptualized a modeling approach for supply chains. This approach has already been utilized in several works, especially in a very sophisticated simulation analysis of quantitative consequences of the bullwhip effect [7].

Evidence for the practical relevance of the bullwhip effect to supply chain management is provided by works on the financial consequences of the bullwhip effect. So there are estimates for the cost of the bullwhip effect available. Based on these estimates the companies should be able to increase their profits – depending on the source – by 8.4 up to 20.1 % [9] or by 10 up to 30 % [10] by avoiding the bullwhip effect.

If efficiency gains are realized in a supply web, a distribution problem arises. The actors cooperating in a supply web know that they are realizing the efficiency gains by mutually coordinating their activities. Besides each actor is interested in maximizing his own gain at the charge of the other actors in the supply web. Thus, supply webs suffer from a built-in conflict between cooperation and defection.

The distribution problem of efficiency gains in supply webs consists in distributing the efficiency gains among partially autonomous actors in a supply web in a manner that the actors accept as fair and thus find it advantageous to cooperate with each other. Otherwise, if it would be advantageous for at least one of the actors to leave the coalition, the supply web would collapse. Thus, a stability requirement for solutions of the distribution problem of efficiency gains is formulated: problem solutions are only regarded as desirable, if they ensure that all actors of a supply web are willing to cooperate with each other. The fulfillment of this stability requirement is generally circumscribed by the actors' acceptance of the distribution of the efficiency gains as fair.

2. The τ-value concept for the fair distribution of efficiency gains in supply webs

2.1 A generic distribution problem for supply webs

In the following, the generic distribution problem to allocate a positive efficiency gain realized through the collaboration in a supply web in a fair manner among the actors of this supply web is discussed. This generic distribution problem is the class of all instances which on the one hand have the basic structure of the generic distribution problem and on the other hand in which each parameter describing the problem is substantiated by numerical values.

The starting point of the distribution problem in conventional game theory is the efficiency gain G to be distributed with $G \in \mathbb{R}_+$ where $\mathbb{R}_+$ is the set of all positive real numbers. The way the efficiency gain G is calculated does not matter, since values are considered as given in game theory. But this is very unsatisfying from a management point of view, since it is not intended to solve a formally specified distribution problem (formal problem). Instead, from a management perspective the real problem of the fair distribution of efficiency gains has the highest priority.

The conceptualization of the real problem requires notions about how to obtain the informations which are assumed to be known in the formal problem. There are two ways to determine quantitatively the efficiency gain G. Firstly, it is possible to fall back on rough estimations of the efficiency gains based on expert opinions. These experts could be managers of the considered supply web. Moreover, third parties like consultants specialized on analyzing supply webs may be considered. Secondly, it is possible to estimate the efficiency gain to be distributed by comparative modelling. This analytic determination of the efficiency gain rests upon information about the supply web for a given number of past periods provided by the controlling department.

Comparative modelling consists of two different approaches which are applied jointly. On the one hand, for each actor who belongs to a supply web it is determined in a partial model how much profit the actor would have made in case of adapting optimally to the information concerning the past periods (partial profit). After that the overall profit can be calculated as the sum of the partial profits of all actors in the supply web. On the other hand, a total model of the supply web is constructed. This model contains all actors of the supply web including all mutual material and information flows. Sophisticated models have been developed for this holistic modelling approach [7, 18; see also 16, 21]. The total model is utilized to determine the profit the actors in the supply web would have made in case of the cooperation of all actors in the supply web. This cooperation includes an optimal coordination of the partial plannings. This total profit of cooperative optimization is compared to the overall profit in case of isolated optimization. If the total profit (cooperative optimization) is greater than the overall profit (isolated optimization), than the difference is the efficiency gain.

From a management point of view the second procedure to determine the probable efficiency gain is to be preferred to the first one, since the determination of the efficiency gain with the formal models and the underlying informations is easily comprehensible and thus can be discussed critically. On the contrary, it is usually hard to discuss rough estimations. Moreover, it may be especially difficult to question the assessments of experts.

The following *four requirements* are considered to be important for the game theoretic modelling of the generic distribution problem: The first requirement is that it is possible to explicate the scope of operation for alternative distribution results that emerge from different assumptions regarding the rationality of the actors. The second requirement is that it is possible to explicate good reasons that make it possible to justify a distribution problem determined by a game theoretic solution concept and thus support the acceptance of the distribution result. The third requirement is that the solution of the distribution problem can be communicated easily in the supply web. The fourth requirement is to ensure the existence and uniqueness of the solution of the distribution problem.

The generic distribution problem to allocate an efficiency gain G with $G\in\mathbb{R}_{+}$ among N actors of a supply web with $N\in\mathbb{N}_{+}$ and $n\geq 2$ can be modelled in a first approach with basic elements of cooperative game theory. But this is only correct as long as it is abstracted from the guideline that good reasons can be stated to accept a distribution of the efficiency gain G among the N actors as fair. Thus, only a reduced form of the above defined generic distribution problem is modelled, since the fairness condition is neglected. At first, this reduced generic distribution problem is regarded to accentuate the relevance of the τ-value solution concept from a management point of view afterwards.

From the viewpoint of cooperative game theory the reduced generic distribution problem for supply webs is to determine a distribution function v to allocate a share v_n of the efficiency gain G to each actor A_n with $n\in\{1,...,N\}$ in the great coalition $K_0 = \{A_1,...,A_N\}$ of all actors in the supply web. If it is additionally assumed that the distribution function must not assign a negative amount as share of the efficiency gain G, then the distribution function v with $\mathbb{R}_{\geq 0}$ as set of all non-negative real numbers can be specified as follows: $v: A \rightarrow \mathbb{R}_{\geq 0}$ with $A_n \mapsto v(A_n) = v_n$ for each actor A_n. The solution of the reduced distribution problem is every N-tupel $\underline{v}$ with $\underline{v} = (v_1,...,v_N)$ which assigns a share v_n of the efficiency gain G to each actor A_n in the set of actors A. This N-tupel is the solution point L_v in the N-dimensional real number space $\mathbb{R}^N_{\geq 0}$. The solution point L_v is represented as column vector $\vec{v} = (v_1,...,v_N)^T$.

The *standard approach* of cooperative game theory to solve a distribution problem like the one specified above consists of two steps.

In the first step a characteristic function is developed. This function refers to all possible coalitions which could be formed of the actors in the considered supply web. Furthermore, “degenerate” coalitions formed by one actor are also feasible. A coalition K_m is a non empty subset of the set $A = \{A_1,...,A_N\}$ of all actors in a supply web ($\varnothing \subset K_m \subseteq A$).

The following holds for the characteristic function c with $\wp$ as power set operator: $c:\wp(A)\rightarrow\mathbb{R}_{\geq 0}$ with $K_m \mapsto c(K_m)$ for each coalition K_m with $\varnothing \mapsto c(\varnothing)=0$. This characteristic function assigns the amount $c(K_m)$ the respective coalition can claim with good reasons to the respective coalition K_m out of the set of actors A. In case of the great coalition $K_0 = A$ this is the overall efficiency gain G: $c(K_0) = G$. For all other coalitions K_m with $\varnothing \subset K_m \subset A$ these are the amounts $c(K_m)$ these coalitions could realize on their own outside the great coalition K_0 and therefore in competition to the rest coalition $K_0 \setminus K_m$.

In the second step the shape of the distribution function v is determined by calculating the distribution function values $v(A_n) = v_n$ for each actor A_n in the set $A = \{A_1,...,A_N\}$ of all actors in the supply web. Only two information sources are considered to calculate the values $v(A_n) = v_n$ for all actors A_n with $n = 1,...,N$. On the one hand, these are the amounts each feasible coalition can claim due to the characteristic function c from the first step. On the other hand, the applied solution concept specifies how the distribution function values $v(A_n) = v_n$ are calculated based on the values $c(K_m)$ of the characteristic function c for all feasible coalitions K_m with $m = 0,1,...,2^N-2$. When all distribution function values $v(A_n) = v_n$ are determined, there is a N-tupel $\underline{v} = (v_1,...,v_N)$ as a solution $\underline{v}$ for the respectively regarded instance of the generic distribution problem in its reduced form.

The aforementioned standard approach of cooperative game theory is not satisfying from a management point of view. The main weakness of this approach lies in the characteristic function c which is assumed to be known in conventional game theoretical analyses. This information premise can be judged as far away from reality, since in practice it is often not known for each feasible coalition K_m which value $c(K_m)$ is reasonably appropriate for the respective coalition.

However, it is conceivable to calculate the values $c(K_m)$ of the characteristic function c by applying calculation procedures a priori. But due to the great number 2^N-1 of feasible coalitions exploding exponentially with the number N of actors in a supply web it has to be assumed at least for great supply webs that the calculation of all values $c(K_m)$ of the characteristic function c fails because of the large calculation effort. For example, if the values $c(K_m)$ are not known from the start, a supply web with $N = 10$ actors requires to accomplish up to $2^{10}-1 = 1.023$ calculation procedures.

From a management point of view a game theoretic solution concept is of great interest which makes it possible to calculate the values v_n of the distribution function v for all actors A_n in a supply web as a solution for the generic distribution problem without having complete knowledge about the characteristic function c. Instead, such a non state of the art solution concept should refer to as few as possible coalitions to calculate the values v_n of the distribution function v for all actors A_n in a supply web. This requirement of *minimal knowlegde* is added to the four above mentioned requirements for the reconstruction of the real problem of the distribution of efficiency gains in supply webs.

2.2 The τ-value solution concept

The τ-value was initially proposed as a solution concept for cooperative games by TIJS in 1980 [11]. Subsequently, this solution concept was further developed, especially by TIJS and DRIESSEN [cf. 2, 3, 4, 5, 12, 13, 14, see also for the τ-value 1, 6, 19, 20]. Up to now, only a few research efforts have tried to apply the τ-value solution concept in the area of economics. However, the corresponding papers are focused on the distribution of fixed costs or overhead

costs [14, 19]. To the knowledge of the authors, the τ-value has not been applied to the solution of the distribution problem of efficiency gains in supply webs until now.
In conventional game theory it is common to introduce new solution concepts by specifying its applicability conditions and its calculation formulas or its calculation algorithms. If the applicability conditions of a solution concept are available in an axiomatized form, it is often argued that the proposed solution concept is the only one that is capable of fulfilling a set of formally specified axiomatic requirements. In this case the solution concept can be justified by fulfilling axioms which are assumed to be "given" or "reasonable". A critical reflection regarding the justification of such axioms can only be found seldomly. Thus, it is rather difficult to detect the connection between some axioms and the respective real problems. However, sometimes it seems as if some axioms are introduced to enforce the application of a certain solution concept which fulfills the axioms.
In the paper at hand, this way of proceeding is not adopted, since it suffers from its formalistic approach to regard a solution concept primarily with respect to its formal applicability and calculation. From a management point of view such a way of proceeding lacks orientation towards the real problem. Instead, it would be desirable to justify a game theoretical solution concept in the following manner: Starting from the real problem to distribute efficiency gains among the actors in a supply web the solution concept should be developed in an easily understandable manner so that it is derivable from plausible requirements oriented towards the real problem. Furthermore, it should be possible to have good reasons to accept the resulting solution result as fair distribution. In the following, the authors try to develop a *justification program* for game theoretic solution concepts oriented towards real problems exemplarily with respect to the τ-value solution concept. A major concern of this paper is to reconstruct the well known τ-value solution concept in a new way with regard to management concerns.
The starting point for distributing an efficiency gain G with $G \in \mathbb{R}_+$ among the N actors of a supply web with $N \in \mathbb{N}_+$ and $n \geq 2$ is the reduced generic distribution problem. The basic idea of the reconstruction of the τ-value solution concept is to restrict the solution space $\mathbb{R}^N_{\geq 0}$ for the generic distribution problem by successively adding five requirements which stem from the real problem of the fair distribution of efficiency gains. The following chain of reasoning yields the τ-value as a "senseful" solution for the generic distribution problem, which is acceptable as fair.
The *first requirement* for a solution concept is the condition of *individual rationality*. This condition assumes that every actor in the supply web acts rationally in sense of the conventional concept of perfect rationality. This means that each actor maximizes her or his individual utility. Moreover, it is assumed that there are no envy effects. That is each actor evaluates her or his individual utility without considering the shares of the other actors. Furthermore, the information processing capacity of the actors is not restricted so that the actors are capable of calculating their individual utility maximum. The condition of individual rationality causes a restriction of the solution space $\mathbb{R}^N_{\geq 0}$, since it would not be rational for an actor A_n to participate in the supply web within the coalition K_0, if this coalition yields a smaller utility in comparison to leaving the coalition and realizing the amount $c(\{A_n\})$ outside the supply web. Thus, the condition of individual rationality can be formulated with the function c as follows:

$$\forall L_v \in \mathbb{R}^N_{\geq 0}: \quad L_v = \begin{pmatrix} v_1 \\ ... \\ v_N \end{pmatrix} \geq \begin{pmatrix} c(\{A_1\}) \\ ... \\ c(\{A_N\}) \end{pmatrix} \tag{1}$$

The *second requirement* is the *efficiency condition*. The efficiency condition postulates that the efficiency gain G is distributed exactly among the coalition $K_0 = \{A_1,...,A_N\}$ of all actors. Thus, the following equation has to hold true with L_v as solution point for a feasible solution $\underline{v} = (v_1,...,v_N)$ of the distribution problem in the solution space $\mathbb{R}^N_{\geq 0}$:

$$\forall L_v \in \mathbb{R}^N_{\geq 0}: \quad L_v = \begin{pmatrix} v_1 \\ ... \\ v_N \end{pmatrix} \rightarrow \sum_{n=1}^{N} v_n = G \tag{2}$$

The efficiency condition implies a further restriction of the solution space $\mathbb{R}^N_{\geq 0}$, since all solutions of the distribution problem, which fulfil the requirement of individual rationality as well as the efficiency condition, are solution points L_v on a hyperplane H in the N-dimensional solution space $\mathbb{R}^N_{\geq 0}$. This hyperplane H is the set of all solutions $\underline{v} = (v_1,...,v_N)$ of the distribution problem, which are fulfilling the restriction $\sum_{n=1}^{N} v_n = G$.
The *third requirement* is the *rationality condition* for *maximum allocable shares of the gain.* This condition has the character of a condition of collective rationality, since it mirrors the rational consideration of all N-1 actors of the so called marginal coalition MK_n with $MK_n = K_0 \backslash \{A_n\} = \{A_1,...,A_{n-1}, A_{n+1},...,A_N\}$ to grant actor A_n at most the share $v_{n.max}$ of the efficiency gain G, so that the efficiency gain G would decrease, if actor A_n would leave the great coalition $K_0 = \{A_1,...,A_N\}$. This rationality condition satisfies the following:

$$\forall n = 1,...,N \ \forall v_n \in \mathbb{R}: v_n \leq v_{n.max} \wedge v_{n.max} = c(K_0) - c(MK_n) = G - c(MK_n) \tag{3}$$

In the solution space $\mathbb{R}^N_{\geq 0}$ the point, in which the maximum allocable share $v_{n.max}$ of the gain is assigned to each actor A_n with n = 1,...,N, is called *upper bound* UB or *ideal point* for the distribution of efficiency gains G.
The *fourth requirement* for the solution concept is a *rationality condition* for *minimal allocable shares of the gain.* This condition has the character of a condition of collective rationality as well, since the condition reflects the rational consideration of all N-1 actors of the marginal coalition MK_n with $MK_n = K_0 \backslash \{A_n\}$, to grant actor A_n at least the share $v_{n.min}$ of the efficiency gain G, so that she or he could believably threaten to found at least one so called outsider coalition $AK_{n.q}$. An outsider coalition is a coalition $AK_{n.q}$ of former actors of the supply web, who are leaving the great coalition $K_0 = \{A_1,...,A_N\}$ at least hypothetically and which has at least the actor A_n as „leader". Moreover, an outsider coalition can never contain all actors of the great coalition K_0, since besides no non-empty residual coalition $RK_{n.q} = K_0 \backslash AK_{n.q}$ would exist anymore, whose actors could generate the efficiency gain to be distributed.
It is important for the solution concept of the τ-value which outsider coalitions $AK_{n.q}$ enable an actor A_n to threaten in a believable manner. Initially, it is assumed that the amount $c(AK_{n.q})$ is known. Then it is premised that actor A_n leads the outsider coalition $AK_{n.q}$ and that she or he offers all other actors an optimal incentive to defect. Actor A_n makes an offer to the other actors in an outsider coalition $AK_{n.q}$. This offer consists of so called side payments and it ensures that the utility of each other actor is the same as his or her maximum utility in the

great coalition K_0. In this case the actors in an outsider coalition have no incentives to remain in the great coalition K_0. The operationalization of the side payments takes place in the following way with the amount $c(\{A_n\} \mid AK_{n.q})$ realizable by actor A_n in the outsider coalition and with the index set $IN_{n.q}$ of all indices of actors in the outsider coalition $AK_{n.q}$:

$$\forall \varnothing \subset AK_{n.q} \subset A: \ \left(A_n \in AK_{n.q} \wedge \{A_n\} \subset AK_{n.q}\right) \rightarrow \ldots$$

$$c\left(\{A_n\} \middle| AK_{n.q}\right) = c\left(AK_{n.q}\right) - \sum_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{m.max} \tag{4}$$

The amounts $c(\{A_n\} \mid AK_{n.q})$ utilized by actor A_n to threaten to found an outsider coalition A_n may be negative. This can be explained due to two reasons. Firstly, the sum $\sum_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{m.max}$ of the side payments can be greater than the amount $c(AK_{n.q})$ realized by the outsider coalition $AK_{n.q}$. In this case the leading actor A_n has to withdraw the partial amount $\sum_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{m.max} - c\left(AK_{n.q}\right)$ from savings or even take on debt. Secondly, if actor A_n is the sole actor in the outsider coalition $AK_{n.q}$ and thus the above mentioned side payments are not required, then the amount $c(\{A_n\})$ may be negative as well. For example, actor A_n may be not competitive without collaborating in the supply web. In both aforementioned cases with $c(\{A_n\} \mid AK_{n.q}) < 0$ a threat would not be believable. Thus, both cases are excluded from the rationality condition for minimal allocable shares of the gain. Summa summarum the complete rationality condition for minimal allocable shares of the gain is as follows:

$$\forall n = 1,\ldots,N \ \forall v_n \in \mathbb{R}_{\geq 0}: \quad v_n \geq v_{n.min} \ \wedge \ v_{n.min} = \max\{c_{n.1}, c_{n.2}, 0\}$$

with:

$$c_{n.1} = \max\left\{ c\left(\{A_n\} \middle| AK_{n.q}\right) = c\left(AK_{n.q}\right) - \sum_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{m.max} \ \middle| \ \ldots \ \varnothing \subset AK_{n.q} \subset A \ \wedge \ \{A_n\} \subset AK_{n.q} \right\} \tag{5}$$

$$c_{n.2} = c\left(\{A_n\} \middle| AK_{n.q}\right) = c\left(\{A_n\}\right) \ \text{for} \ AK_{n.q} = \{A_n\}$$

The *lower bound* LB for the distribution of the efficiency gain G is that point in the solution space $\mathbb{R}^N_{\geq 0}$, in which the minimal allocable share $v_{n.min}$ of the gain is assigned to each actor A_n with $n = 1,\ldots,N$. The lower bound LB is often called *threat point,* especially in the literature on the τ-value it is called minimal right vector [11].

The following *integrity condition* for the ratio between the lower bound LB and the upper bound UB for the shares of the efficiency gain to be distributed as well as the hyperplane H for the compliance with the efficiency condition is introduced to avoid some particular complications, which cannot be discussed here because for space reasons:

$\forall L_v, LB, UB \in \mathbb{R}^N \; \forall G \in \mathbb{R}_+ :$

$$\left(L_v = \begin{pmatrix} v_1 \\ \dots \\ v_N \end{pmatrix} \wedge \; LB = \begin{pmatrix} v_{1.min} \\ \dots \\ v_{N.min} \end{pmatrix} \wedge \; UB = \begin{pmatrix} v_{1.max} \\ \dots \\ v_{N.max} \end{pmatrix} \right) \tag{6}$$

$$\rightarrow \left(\sum_{n=1}^{N} v_{n.min} \le \sum_{n=1}^{N} v_n = G \le \sum_{n=1}^{N} v_{n.max} \wedge \; LB \le UB \right)$$

The *fifth requirement* for the solution concept is that the concept has to fulfill an *operational fairness criterion*: The greater bargaining power of an actor A_n is, the greater is her or his share v_n of the efficiency gain G.

The *bargaining power* of an actor A_n is affected by two opposed effects:

- On the one hand the bargaining power of an actor A_n is measured by the contribution the actor would make, if she or he would take part in the marginal coalition MK_n with $MK_n = K_0 \backslash \{A_n\}$ and thus would make this marginal coalition a great coalition K_0. This *positive network effect* is the maximum allocable share $v_{n.max}$ of the gain in above mentioned third requirement.
- On the other hand the bargaining power of an actor A_n is measured by her or his threat potential build up of the believable threat to found at least one outsider coalition $AK_{n.q}$. This *negative network effect* has been specified as minimal allocable share of the gain $v_{n.min}$ in the fourth requirement above.

The *good reasons* to accept the distribution of the efficiency gain G among the actors A_n in a supply web and therefore a feasible solution L_v of the generic distribution problem as fair can be specified as follows: It is regarded as *fair* to grant actor A_n a share v_n of the efficiency gain G which is positively correlated with the actors' contribution to build the great coalition (positive network effect) and with the threat potential to prevent that the great coalition comes into existence (negative network effect).
The aforementioned characterization of a fair distribution of the efficiency gain is mainly qualitative. Thus, this characterization offers some scope of interpretation regarding the numerical determination of the shares v_n for all actors A_n in a supply web. Therefore, a *quantification of the fairness criterion* in form of a calculation rule for τ-value is required. This calculation rule should be as easy as possible to understand to gain acceptance in practice. The calculation rule employed is the following new type of *τ*-value formula:

$\forall n = 1, \dots, N :$

$$v_{n.\tau} = \begin{cases} \alpha \bullet \dfrac{v_{n.max}}{\sum_{n=1}^{N} v_{n.max}} \bullet G + \beta \bullet \dfrac{v_{n.min}}{\sum_{n=1}^{N} v_{n.min}} \bullet G \; ; \text{ if } \sum_{n=1}^{N} v_{n.max} \ne \sum_{n=1}^{N} v_{n.min} \\ v_{n.max} = v_{n.min} \; ; \text{ if } \sum_{n=1}^{N} v_{n.max} = \sum_{n=1}^{N} v_{n.min} \end{cases} \tag{7a}$$

with:

$$\alpha = \frac{G - \sum_{n=1}^{N} v_{n.min}}{\sum_{n=1}^{N} v_{n.max} - \sum_{n=1}^{N} v_{n.min}} \bullet \frac{\sum_{n=1}^{N} v_{n.max}}{G}$$

$$\beta = \frac{\sum_{n=1}^{N} v_{n.max} - G}{\sum_{n=1}^{N} v_{n.max} - \sum_{n=1}^{N} v_{n.min}} \bullet \frac{\sum_{n=1}^{N} v_{n.min}}{G} \qquad (7b)$$

Firstly, this calculation rule is characterized by capturing the bargaining power of the actor by two summands. The first summand reflects the bargaining power of actor A_n due to her or his contribution $v_{n.max}$ to the coalition (positive network effect) by means of the share $v_{n.max} \bullet G$ of the efficiency gain. The second summand represents the bargaining power of actor A_n due to her or his threat potential $v_{n.min}$ to dissolve the great coalition (negative network effect) by means of the share $v_{n.min} \bullet G$ of the efficiency gain.
Secondly, the contribution and the threat potential of actor A_n are not measured absolutely, but are relativized with respect to the sums of the contributions and the threat potentials, respectively. This is a normalization of the contribution and of the threat potential of actor A_n regarding the upper bound UB and the lower bound LB, respectively.
Thirdly, the contribution and the threat potential of actor A_n are weighted with α and β. This weighting allows to represent the τ-value in a more compact form compared to the linear combination of the upper bound (ideal point) and the lower bound (threat point) lying on the hyperplane H within the solution space satisfying the efficiency condition (see figure 1 on next page):

$$\forall n = 1,...,N: \quad v_{n.\tau} = \gamma \bullet v_{n.max} + (1-\gamma) \bullet v_{n.min} \qquad (8)$$

with:

$$\gamma = \begin{cases} \dfrac{G - \sum_{n=1}^{N} v_{n.min}}{\sum_{n=1}^{N} v_{n.max} - \sum_{n=1}^{N} v_{n.min}} & ; \text{ if } \sum_{n=1}^{N} v_{n.max} \neq \sum_{n=1}^{N} v_{n.min} \\ 0 \vee 1 & ; \text{ if } \sum_{n=1}^{N} v_{n.max} = \sum_{n=1}^{N} v_{n.min} \end{cases}$$

The τ-value is a *compromise solution* for the distribution problem. This compromise solution is characterized by two properties. Firstly, the compromise solution is *Pareto efficient* in terms of the aforementioned efficiency condition. Secondly, the compromise solution represents the *simplest* compromise between the *ideal point* and the *threat point*. Between these points no simpler connection can be constructed in the solution space $\mathbb{R}^N_{\geq 0}$ than the direct rectilinear distance determined by equation (8). The characterization of the τ-value as a compromise solution for the distribution problem yields a further *good reason* to accept the distribution of the efficiency gains as fair, since intuitive preconceptions about what is accepted as fair contain the normative connotation that fair distributions should be based on a compromise between the interests of the involved actors. In the τ-value solution concept these interests are operationally specified with the aid of the ideal point and the threat point.

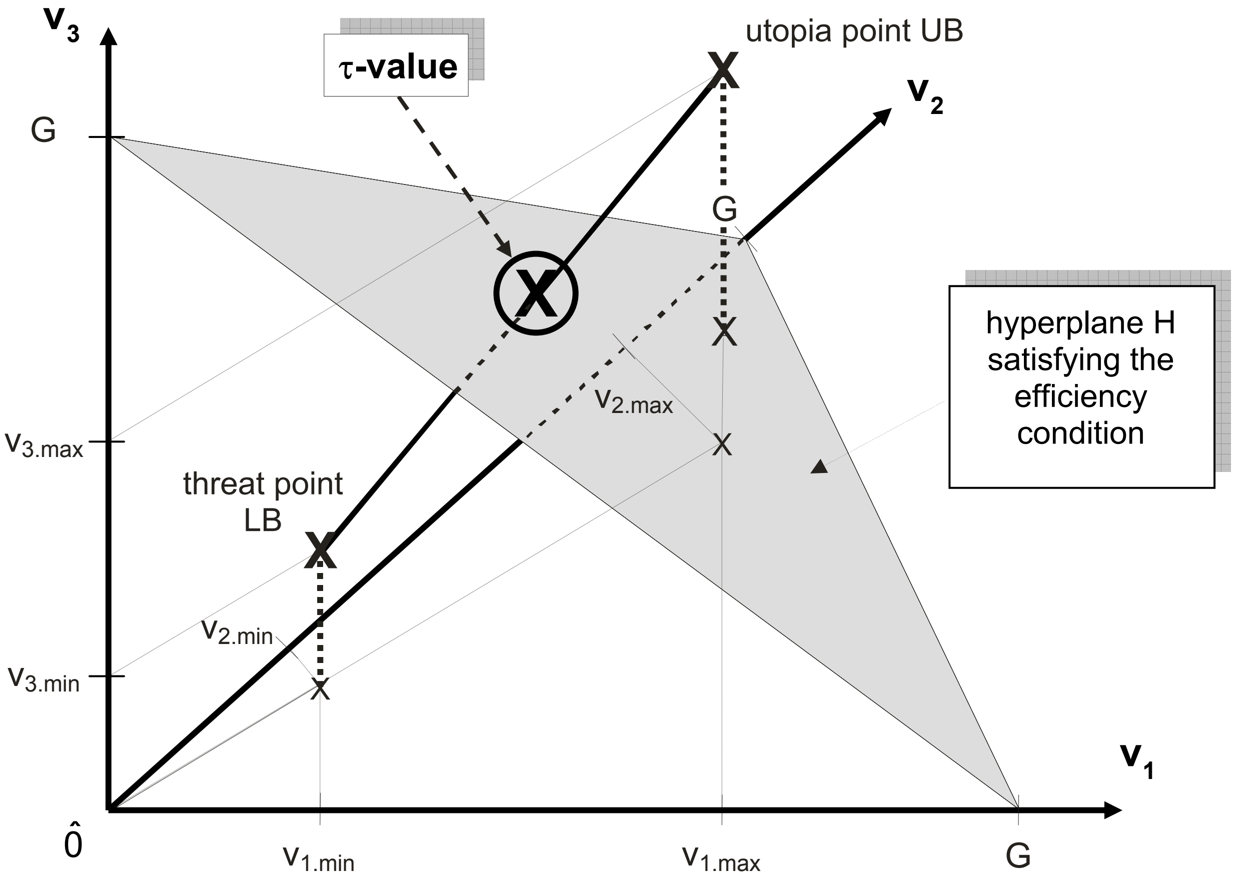

Figure 1. the τ-value

It can be shown that for each instance of the generic distribution problem which fulfils the integrity condition (6) exactly one solution exists that is satisfying the above mentioned requirements of individual and collective rationality as well as the requirements of efficiency and of the fairness of efficiency gain distribution. This solution is the τ-value which can be determined by one of the equivalent calculation rules (7a and b) versus (8).

3. Conclusions

This paper has presented the τ-value as a solution concept for the generic distribution problem. This distribution problem consists of allocating the efficiency gain among the actors of a supply web in a way that makes it possible to state good reasons for the fairness of the distribution of the efficiency gain. Five requirements were developed as evaluation criteria to assess the suitability of the τ-value solution concept for the distribution of efficiency gains in supply webs.

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