A combined Goal Programming and Analytic Hierarchy Process (AHP) approach for efficiency analysis of production orders

TORBEN HUEGENS; MALTE L. PETERS; STEPHAN ZELEWSKI

1 Introduction: problem identification

Practitioners often have to analyze the efficiency of production orders considering multiple inputs and outputs. On the one hand, sophisticated techniques like *Data Envelopment Analysis (DEA)* (Charnes, Cooper, and Rhodes 1978; Charnes, Cooper, and Thrall 1991; Ramanathan 2003; Cooper, Seiford, and Zhu 2004; Kleine 2004) and *Operational Competitiveness Rating (OCRA)* (Parkan and Wu 1998) can be utilized to analyze the efficiency. Nevertheless, practitioners often cannot cope with the difficulties associated with the premises of those techniques. Thus, this paper provides a simple approach. On the other hand, simple techniques like *Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)* (Hwang and Yoon 1981) can also be employed. However, all those techniques suffer from a major drawback: They are not capable of considering two sided-goals. In practice, however, it is often desired to meet a specific value for a specific criterion. *Goal Programming (GP)* offers the opportunity to consider this requirement of two sided-goals.

To address these problems, a combined Goal Programming and *Analytic Hierarchy Process (AHP)* approach is proposed in this paper. Normally, the application of Goal Programming requires the decision maker to establish the target values (goals) as well as the realized or expected actual values of the production orders. The approach proposed in this paper obtains the target values of lower one-sided goals in a self-acting way by deriving them from the actual values. Then an efficient frontier can be constructed on the basis of these target values, which could be interpreted as a virtual production order.

In section 2 terminological preliminaries as well as methodological preliminaries are outlined. The following section 3 presents the goal programming model incorporating the AHP data. Section 4 concludes the paper with some remarks.

2 Preliminaries

2.1 Terminological preliminaries

Normally, efficiency is defined as the ratio between the achieved outputs and the employed inputs. The efficiency of a production order can be evaluated, if a standard of comparison is known. In case of absolute efficiency, this standard of comparison is given by a production function as the efficient frontier of the technology set. If the technology set is unknown, only a comparison of the production orders among each other is possible. This type of efficiency is called relative efficiency (e.g. Dyckhoff and Allen 1999, p. 415). In case of relative efficiency, the technology set can be partially constructed out of the considered production orders.

The decision maker, however, has to determine, which criteria have to be considered. Fundamentally, in the AHP cost criteria – like material costs and energy costs – and benefit criteria – like the contribution margin of the production order – can be differentiated. If cost criteria (benefit criteria) are evaluated, the most desired criterion value is the minimum (maximum) value. Inputs are modelled as cost criteria, since low (high) input quantities – *ceteris paribus* – positively (negatively) influence the efficiency of a production order. Outputs are regarded as benefit criteria, since high (low) output quantities are – ceteris paribus – positively (negatively) influencing the efficiency of a production order. This is only valid for desired products. For undesired products – like emissions, waste, wastewater, and lost heat – inputs (outputs) are modelled as benefit criteria (cost criteria). High input quantities of undesired products have a positive influence on the efficiency of a production order; while high output quantities have a negative influence on the efficiency.

2.2 Methodological preliminaries

2.2.1 Analytic Hierarchy Process

The goal programming model presented in section 3 requires several assessments such as the assessment of criteria importance weights and the assessment of production orders regarding several criteria. These assessments can be undertaken using the Analytic Hierarchy Process (AHP) (e.g. Saaty 2001; Saaty and Vargas 1994; Saaty 1994a; Saaty 1994b).
The standard AHP is a well-known multi-criterion decision making technique, developed by Saaty. The goal is to find a best rational and intuitive selection out of different alternatives.
A typical decision problem is structured in three levels. The goal of the AHP stands at the top of the hierarchy, while the criteria are placed on the second level. The criteria are used to judge the alternatives on the lowest level. In addition, the local preference of one criterion in comparison to another is evaluated to find the final decision about the goal.
The AHP requires pair wise comparison judgments concerning the dominance of one element (e.g. criteria and alternatives) over another using a 1-9 scale, in order to obtain the intensity of importance for each element:

intensity of importance	definition
1	equal importance
3	moderate importance
5	strong importance
7	very strong importance
9	extreme importance
2, 4, 6, 8	for compromises between the above values
$\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7},\frac{1}{8},\frac{1}{9}$	if element i has one of the above nonzero numbers assigned to it when compared with element j, then j has the reciprocal value when compared with i

Table 1: Intensity of importance scale (e.g. Saaty 1994b, p. 26; Saaty, and Vargas 1994, p. 6)

The pair wise comparison judgments are entered in a square matrix A. If an element i is judged to be moderately important by comparison with another element j, for example, a 3 is entered as the value for the pair wise comparison judgment a_{ij} in the matrix A, while the reciprocal value is entered for the complementary comparison judgment a_{ji}.

$$A = \begin{pmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{i1} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{nj} & \dots & a_{nn} \end{pmatrix} \quad \text{with} \quad \begin{array}{l} \forall i = 1,\dots,n; \forall j = 1,\dots,n : a_{ij} > 0 \\ \forall i = j : a_{ij} = 1 \\ \forall i = 1,\dots,n; \forall j = 1,\dots,n : a_{ij} = a_{ji}^{-1} \end{array}$$

The importance weights are derived by computing a normalized eigenvector (priority vector) of the matrix A. The computing of the normalized eigenvector can be done using approximate methods, if the dimension of the matrix is smaller than or equal to three (e.g. Saaty 2001, p. 55). If the dimension is larger than three, the so-called “Power-Method” or another exact method has to be used to generate accurate normalized eigenvectors (e.g. Saaty 2001, pp. 55). The priorities for every criterion and every alternative can be derived by computing the normalized eigenvector. In the following, the global priorities of the criteria are calculated by the multiplication of the local priorities along all paths from the top to the bottom of the criteria hierarchy. Afterwards, these global priorities are multiplied by the local priorities for every alternative. The overall priorities are obtained by summing up these products. Based on the overall priorities the decision maker can decide which alternative is the most valuable. For the combined Goal Programming and AHP approach proposed here the calculation of the overall priorities is not necessary.

In doing the pair wise comparison judgments there is the possibility that inconsistencies in the matrices occur, due to the subjectivity of the values in matrix A (e.g. Saaty 1994a, p. 438). To identify these inconsistencies the consistency index (C.I.) and the consistency ratio (C.R.) can be calculated. The C.I. and C.R. use two characteristics of matrix A: In matrix A exists at least one eigenvalue λ with an according eigenvector υ (e.g. Saaty 1994b, p. 41). The maximum eigenvalue λ_{max} is equal to the dimension n of matrix A, if all pair wise comparison judgments are consistent. The eigenvector υ is defined for every eigenvalue λ as:

$$A * \upsilon = \lambda * \upsilon \text{ with } \lambda \in \Re \text{ and } \upsilon \neq 0$$

The maximum eigenvalue λ_{max} of matrix A is larger than its dimension, if matrix A is not consistent. To compute the C.I., λ_{max} has to be derived.

$$C.I. = \frac{\lambda_{max} - n}{n - 1}$$

The value of the C.I. is compared with the C.I. of a randomly filled matrix (the consistency index for these types of matrices is called Random Index (R.I.); e.g. Saaty 2001, p. 65 and p. 84):

$$C.R. = \frac{C.I.}{R.I.}$$

If the value of C.R. is larger than 0.10, it is recommended to improve the matrix consistency (e.g. Saaty 1994b, pp. 27 and p. 42).
A major problem of the AHP is making pair wise comparison judgments in a model that has a large number of elements (e.g. criteria, production orders). This can be very time and resource consuming. Accordingly, if a large number of criteria is considered, it could be more practical to create a hierarchy of these criteria. While a hierarchy of the elements can be used to reduce the number of pair wise comparison judgments, it can also be helpful to have well-structured criteria. Further refinements of the AHP are addressed in the specialized literature (e.g. Saaty 2001). A second problem is called rank-reversal (e.g. Saaty 2001, pp. 25 and pp. 42). If a new alternative is added to the decision problem, the ranks of the "old" alternatives can change. A third problem valid not only for the AHP is that it is partly a subjective technique, because the pair wise comparison judgments are made by individual decisions about elements. Nevertheless, the AHP has the advantage, that the use of mathematical techniques, like computing eigenvectors and performing well-defined normalizations is more "objective" than other techniques, like e.g. simple scoring techniques. Another problem is the equality of quantitative and qualitative criteria in the AHP. In the AHP, qualitative and quantitative criteria can be judged equally with the same ordinal scale. Therefore, there is an information loss, if a quantitative criterion is transformed into a qualitative one transforming the cardinal values to ordinal values. To minimize this problem, Saaty has developed several techniques for the transformation: The first one is called relative measurement (e.g. Saaty 2000, p. 9; Saaty 2001, p. 136). Using relative measurement quantitative and qualitative criteria are handled equally. The pair wise comparison judgment is done as for qualitative criteria assigning a value of the above-mentioned intensity of importance scale. A major deficit of this technique is the information loss. For example, if the costs of two production orders are known exactly, e.g. 32,500 € for production order 1 and 33,000 € for production order 2, and production order 1 is judged to be moderately better than production order 2 (e.g. $a_{12} = 3$), the initial cardinal information about production order costs is lost. The second technique is called absolute measurement (e.g. Saaty 2000, p. 22; Saaty 2001, p. 136). This technique can be differentiated into three sub-types. The first sub-type uses intensities derived from natural language or intensity intervals of cardinal values (e.g. Saaty 2001, pp. 136). These intensities are transformed into the intensity of importance scale ranging from 1 to 9. The second sub-type uses a cardinal-scaled function, with a right-clear connection between the values of a quantitative criterion and the preference of a decision maker for the value of the criterion. The third sub-type is also called direct measurement (e.g. Meixner and Haas 2002, pp. 158). It is a precise type of judgment and causes no information loss. Nevertheless, it can only be used if all values of a criterion for each alternative are known exactly and the decision makers' preferences are linear. In addition, two formulas for the computation of the priorities must be differentiated. If a high value of a criterion is desired, the following formula has to be used for calculating the priority p_i of an alternative i (k_i: value of the alternative i with respect to the considered criterion; n: number of all alternatives):

$$p_i = \frac{k_i}{\sum_{i=1}^{n} k_i} \quad \forall i = 1,...,n$$

In addition, if a low value of a criterion is desirable, the following formula holds:

$$p_i = \frac{\frac{1}{k_i}}{\sum_{i=1}^{n} \frac{1}{k_i}} \quad \forall i = 1,...,n$$

If it is possible to aggregate several criteria l because of the same dimension (so-called clustering), the following formulas can be used for calculating the aggregated priority p_i^a of an alternative i (L: number of all criteria):

$$p_i^a = \frac{\sum_{l=1}^{L} k_{i,l}}{\sum_{i=1}^{n}\sum_{l=1}^{L} k_{i,l}} \quad \begin{array}{l} \forall i = 1,...,n \\ \forall l = 1,...,L \end{array}$$

If a low value of a criterion is preferable, the reciprocal values are used:

$$p_i^a = \frac{\frac{1}{\sum_{l=1}^{L} k_{i,l}}}{\sum_{i=1}^{n} \frac{1}{\sum_{l=1}^{L} k_{i,l}}} \quad \begin{array}{l} \forall i = 1,...,n \\ \forall l = 1,...,L \end{array}$$

All the above mentioned transformation techniques solve a part of the first mentioned problem: The huge amount of possible pair wise comparison judgments is reduced, so the consumption of time and other resources is smaller and the use of the AHP becomes more efficient.

For the efficiency analysis of production orders and the essential judgments of criteria for the goal programming approach, the AHP has four major benefits: Firstly, the problems of transforming qualitative, natural-language pair wise comparison judgments into numerical values can be easily resolved using the intensity of importance scale. Secondly, the subjectivity is limited to the pair wise comparison judgments. After this step, there is no more subjectivity necessary in using the AHP. Thirdly, the AHP is flexible in usage for different areas like e.g. for decision-support problems, efficiency analysis or usage in the information sciences area (e.g. Riedl 2005, pp. 204). The fourth and in this case most important advantage is that it is possible to include qualitative and quantitative criteria into the decision hierarchy of the AHP.

2.2.2 Goal Programming

Charnes and Cooper (1961, p. 215) introduced the goal programming approach. It is a mathematical programming technique designed to handle multiple, possibly conflicting objectives; Tamiz, Jones, and Romero (1998) provide an overview. In goal programming, the desired

level of fulfillment of each objective is viewed as a goal. The technique enables a decision maker to consider one-sided goals and two-sided goals. If the objective is to reach or exceed a one-sided goal, it is called a lower one-sided goal; otherwise, if the objective is to reach or fall below a one-sided goal, it is called an upper one-sided goal. If the objective is to meet a goal as closely as possible, it is called a two-sided goal (e.g. Hillier and Lieberman 2001, p. 332).
The aim of the application of goal programming is to minimize the deviations from the goals considered. So-called deviational variables measure the amount by which the values delivered by the solution of a goal programming model deviate from the respective goal. If a lower (an upper) one-sided goal is considered, the objective function will contain a non-negative under-achievement (overachievement) variable (e.g. Kwak, and Lee 1997, 131). If a two-sided goal is considered, the objective function will contain both an underachievement and an over-achievement variable. Further, the basic goal programming model can be enhanced by considering differences in the relative importance of goals. This enhanced approach is named weighted goal programming and assigns importance weights to the underachievement or overachievement variables according to their relative importance.

3 Model formulation

3.1 Preparation of input data

The goal programming model requires the exogenous assessment of the relative goal importance weights. Prima facie, this could be felt as a disadvantage in comparison to DEA because in DEA the weights are determined model endogenously. Nevertheless, especially practitioners find this strange and mathematically demanding. Therefore, the DEA approach is not chosen. For the sake of acceptance in business practice and ease of communication, an alternative approach is developed by combining goal programming with AHP.
The parameters α_i and β_i enable the decision maker to determine the type of each goal i as a lower one-sided, an upper one-sided or a two-sided goal. The relative goal importance weights w_i are assessed employing the relative measurement mode of the AHP. If all criteria are assessed employing the AHP, they can be modeled as lower one-sided goals, since the most desirable production order is the one with the highest AHP-priority. This is the standard case of the model proposed in this paper. Characteristically, goal programming requires the decision maker to establish the target values (goals). If all criteria are assessed employing the AHP, this step of establishing target values can be omitted by developing a best-of-breed-frontier out of the decision makers' judgments about the production orders. A priority vector $\vec{r}_i$ according to a goal i contains the judgments about the values r_{ij} of all production orders $j = 1,...,J$ with respect to this goal. For each goal i a target value g_i is established by determining the maximum value of the components r_{ij} of the vector $\vec{r}_i$. The resulting vector $\vec{r}$ with I elements g_i represents the best-of-breed-frontier. Analogous to the Data Envelopment Analysis (DEA) where an efficiency frontier is interpreted as virtual Decision Making Unit (DMU) (e.g. Griffin and Kvam 1999, p. 403; Lozano and Villa 2004, pp. 149) the best-of-breed-frontier composed of the I elements g_i for each goal respectively can be interpreted as a vir-

tual alternative. If all criteria are assessed employing the AHP, all goals are modeled as lower one-sided goals $(\alpha_i = 1 \wedge \beta_i = 0)$. If some of the criteria considered are not assessed employing the AHP, they could also be modeled as upper one-sided goals or two-sided goals. Examples for those types of goals are target costs and delivery dates, respectively. This might be the case if practitioners find the AHP-based assessments to time and resource consuming. In this case the parameters α_i and β_i have to be set to the values $\alpha_i = 0 \wedge \beta_i = 1$ and $\alpha_i = 1 \wedge \beta_i = 1$, respectively.

In the following figure all variables needed to solve the goal programming model are listed.

---

model exogenous variables (parameters):

I	number of goals $i = 1,...,I$ with $I \in \mathbb{N}_+$
J	number of production orders $j = 1,...,J$ with $J \in \mathbb{N}_+$
a_{ij}	actual value of production order j for criterion/goal i
a_{ij}^N	normalized actual value of production order j for criterion/goal i
b_{jl}	required capacity of type l for production order j
g_i	target value for goal i, is an exogenous variable, if the values of criterion i are obtained without employing the AHP
M	number of types of capacities $M \in \mathbb{N}_+$
w_i	relative importance of goal i
r_{ij}	value of production order j for criterion/goal i
$\vec{r}_i$	vector of values for criterion/goal i for all production orders $j = 1,...,J$
α_i, β_i	parameters to determine the type of goal, with the following definition:

$\alpha_i = 1 \wedge \beta_i = 1,$ if goal i is a two-sided goal
$\alpha_i = 1 \wedge \beta_i = 0,$ if goal i is a lower one-sided goal
$\alpha_i = 0 \wedge \beta_i = 1,$ if goal i is an upper one-sided goal

model endogenous variables:

d_i^-, d_i^+	lower/upper deviational variables
g_i	target value for goal i, is an endogenous variable, if criterion i is assessed employing the AHP
g_i^N	normalized target value for goal i
x_j	decision variables, with the following definition:

$x_j = 0$: production order j is not selected for execution
$x_j = 1$: production order j is selected for execution

Figure 1: Model variables

3.2 The goal programming model

In the goal programming model proposed here the selection of the most efficient production orders is intended. Figure 2 illustrates the goal programming model. The objective function incorporates the AHP-based relative importance weights w_i as well as the deviational variables d_i^+, d_i^- and the binary variables α_i, β_i to determine the types of goals.

Objective function

$$\text{MIN} \sum_{i=1}^{I} w_i * (\alpha_i * d_i^- + \beta_i * d_i^+)$$

subject to the constraints:

[1] $\sum_{j=1}^{J} a_{ij}^N * x_j + d_i^- - d_i^+ = g_i^N \qquad \forall i = 1,...,I$

[2] $d_i^-, d_i^+ \geq 0 \qquad \forall i = 1,...,I$

[3] $g_i = \underset{j=1,...,J}{\text{MAX}} \{ r_{ij} : r_{ij} \in \vec{r}_i \} \qquad \forall i = 1,...,I \,|\, (\alpha_i = 1 \wedge \beta_i = 0)$

[4] $a_{ij}^N = \dfrac{a_{ij}}{g_i} \qquad \forall i = 1,...,I \quad \forall j = 1,...,J$

[5] $g_i^N = \dfrac{g_i}{g_i} = 1 \qquad \forall i = 1,...,I$

[6] $0 < w_i \leq 1 \qquad \forall i = 1,...,I$

[7] $\alpha_i \in \{0;1\} \wedge \beta_i \in \{0;1\} \qquad \forall i = 1,...,I$

[8] $\neg (\alpha_i = 0 \wedge \beta_i = 0) \qquad \forall i = 1,...,I$

[9] $x_j \in \{0;1\} \qquad \forall j = 1,...,J$

[10.1] $\sum_{j=1}^{J} b_{j1} * x_j \leq b_1$

...

[10.M] $\sum_{j=1}^{J} b_{jM} * x_j \leq b_M$

Figure 2: Goal programming model

The underachievement variables d_i^- and the overachievement variables d_i^+ guarantee that constraint [1] can always be fulfilled. From another point of view, constraint [1] ensures in connection with constraint [2] that the values of the deviational variables d_i^- and d_i^+ are implicitly (model endogenously) determined. The "technical" non-negativity constraint [2] prevents the underachievement variables d_i^- and the overachievement variables d_i^+ from becoming negative. Constraint [3] guarantees that for each lower one-sided goal i a target value g_i is established by determining the maximum value of each vector $\vec{r}_i$. Constraints [4] and [5] are required if some of the goals considered are not assessed employing the AHP. These constraints normalize the values of the criteria that are measured on substantially different scales. Constraint [6] defines the range of possible values for the AHP-based relative importance weights w_i. The binary constraint [7] defines α_i, β_i as control variables that have to be determined by the model designer for the goal type in accordance with their interpretations mentioned above in figure 1. Constraint [8] rules out the not admissible case that a goal is neither a lower one-sided nor an upper one-sided nor a two-sided goal. Moreover, constraint [9] defines the decision variables x_j as binary variables. The decision variable x_j takes the value 1 if a production order j is selected and the value 0 if the production order is not selected. The constraints [10.1] to [10.M] are capacity constraints. They ensure that the number of the most efficient production orders, which are selected for execution, is determined model endogenously.

4 Concluding remarks

This paper has presented a computer-based model for the efficiency analysis of production orders. The solution of this model delivers the group of the most efficient production orders. It integrates two well-known methods, Goal Programming and AHP.

The goal programming model presented in Figure 2 can be solved using a software package for solving optimization problems such as Lingo (2005). Moreover, in order to facilitate the assessments, professional AHP-software like Expert Choice (2005) can be utilized.

However, the practical application of the goal programming model has some shortcomings. An obvious problem is that some of the input data may be fuzzy. For example, a decision maker may be uncertain about the criteria values. For that case, enhanced goal programming models have been proposed using the concept of fuzzy sets (e.g. Martel, and Aouni 1998; Pal, and Moitra 2003). Therefore, future research should be directed towards developing fuzzy goal programming models for the efficiency analysis of production orders.

Literature

Charnes, A.; Cooper, W. W. (1961): Management Models and Industrial Applications of Linear Programming, Volume I. New York – London – Sydney 1961.

Charnes, A.; Cooper, W.; Rhodes, E. (1978): Measuring the efficiency of decision making units. In: European Journal of Operational Research, Vol. 2 (1978), No. 6, pp. 429-444.

Charnes, A.; Cooper, W. W.; Thrall, R. M. (1991): A Structure for Classifying and Characterizing Efficiency and Inefficiency in Data Envelopment Analysis. In: Journal of Productivity Analysis, Vol. 2 (1991), No. 3, pp. 197-237.

Cooper, W. W.; Seiford, L. M.; Zhu, J. (Eds.) (2004): Handbook on Data Envelopment Analysis. Boston 2004.

Dyckhoff, H.; Allen, K. (1999): Theoretische Begründung einer Effizienzanalyse mittels Data Envelopment Analysis (DEA). In: Schmalenbachs Zeitschrift für betriebswirtschaftliche Forschung, Vol. 51 (1999), No. 5, pp. 411-436.

Expert Choice, Inc. (2005): Expertchoice 11. Arlington 2005.

Griffin, P. M.; Kvam, P. H. (1999): A Quantile-Based Approach for Relative Efficiency Measurement. In: Managerial and Decision Economics, Vol. 20 (1999), No. 8, pp. 403-410.

Hillier, F. S.; Lieberman, G. J. (2001): Introduction to Operations Research. 7th ed., Boston – Burr Ridge – Dubuque et al. 2001.

Hwang, C.-L.; Yoon, K. (1981): Multiple Attribute Decision Making – Methods and Applications – A State-of-the-Art Survey. Berlin – Heidelberg – New York 1981.

Kleine, A.: A general model framework for DEA. In: Omega, Vol. 32 (2004), No. 1, pp. 17-23.

Kwak, N. K.; Lee, C. (1997): A Linear Goal Programming Model for Human Resource Allocation. In: Journal of Medical Systems, Vol. 21 (1997), No. 3, pp. 129-140.

Lindo Systems Inc. (2005): Lingo 9.0. Chicago 2005.

Lozano, S.; Villa, G. (2004): Centralized Resource Allocation Using Data Envelopment Analysis. In: Journal of Productivity Analysis, Vol. 22 (2004), No. 1-2, pp. 143-161.

Martel, J. M., Aouni, B. (1998): Diverse Imprecise Goal Programming Model Formulations. In: Journal of Global Optimization, Vol. 12 (1998), No. 2, pp. 127-138.

Pal, B. B.; Moitra, B. N.: A goal programming procedure for solving problems with multiple fuzzy goals using dynamic programming. In: European Journal of Operational Research, Vol. 144 (2003), No. 3, pp. 480-491.

Parkan, C.; Wu, M.-L. (1998): Process selection with multiple objective and subjective attributes. In: Production Planning & Control, Vol. 9 (1998), No. 2, pp. 189-200.

Ramanathan, R. (2003): An Introduction to Data Envelopment Analysis – A Tool for Performance Measurement. New Delhi – Thousand Oaks – London 2003.

Riedl, R. (2005): Der Analytic Hierarchy Process: Ein geeignetes Verfahren für komplexe Entscheidungen in der Wirtschaftsinformatik? In: HMD Praxis der Wirtschaftsinformatik, Vol. 42 (2005), No. 246, pp. 104-114.

Saaty, T. L. (1994a): Highlights and critical points in the theory and application of the Analytic Hierarchy Process. In: European Journal of Operational Research, Vol. 74 (1994), No. 3, pp. 426-447.

Saaty, T. L. (1994b): How to Make a Decision: The Analytic Hierarchy Process. In: Interfaces, Vol. 24 (1994), No. 6, pp. 19-43.

Saaty, T. L. (2000): The Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process. 2nd ed., Pittsburgh 2000.

Saaty, T. L. (2001): Decision Making with Dependence and Feedback: The Analytic Network Process. 2nd ed., Pittsburgh 2001.

Saaty, T. L.; Vargas, L. G. (1994): Decision Making in Economic, Political, Social and Technological Environments: The Analytic Hierarchy Process. Pittsburgh 1994.

Tamiz, M.; Jones, D.; Romero, C. (1998): Goal programming for decision making: An overview of the current state-of-the-art. In: European Journal of Operational Research, Vol. 111 (1998), No. 3, pp. 569-581.