A formal model for production order selection considering synergies

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Abstract

Sometimes, an enterprise may have a great number of potential production orders. On the one hand it might have a lot of customer requests (production to order). On the other hand there might be a great number of production orders based on market forecast (production for stock). Occasionally, it might not be possible to execute all production orders because of capacity constraints. Therefore, a subset of the potential production orders has to be selected for execution.

It is necessary to determine the capacity requirements of each production order. If different resources are required for production order execution, the capacity required of each resource for each production order has to be determined. Apart from this, at least one criterion is required to assess the relative preference of each production order. Here the contribution margin of a production order is chosen as criterion.

Synergies affect the relative preference of production orders. Synergies emerge when fixed costs of resources or fixed-step costs (e.g. set-up costs) are divided among several production orders, resulting in an increase of the contribution margins of the production orders. Such synergies are often neglected in the literature on production order selection.

This contribution presents a formal model for production order selection considering synergies between production orders.

1 Introduction

Sometimes, an enterprise may have a great number of potential production orders. On the one hand it might have a lot of customer requests (production to order). On the other hand there might be a great number of production orders based on market forecast (production for stock). Occasionally, it might not be possible to execute all production orders because of capacity constraints. Therefore, a subset of the potential production orders has to be selected for execution.

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Synergies affect the relative preference of production orders. Synergies emerge, when fixed costs of resources (e.g. tool costs) or fixed-step costs (e.g. set-up costs) are divided among several production orders, so that the contribution margins of the production orders are increased. Another example for a synergy is a sales increase caused by cross selling of products. Such synergies are often neglected in the literature on production order selection. This contribution presents a formal model for the *production order selection problem* considering synergies between production orders.

2 Model formulation

One approach for production order selection is based on knapsack models[1]. This model type makes it possible to consider the contribution margins of the production orders and a capacity constraint. Other applications of the knapsack problem are also possible, such as project selection under a budget constraint, storing a warehouse to maximum value, or package selection for shipment under a space constraint[2].

In the knapsack problem a hiker chooses among several items (production orders) to place in a knapsack. Each item has a certain value (contribution margin of a production order) and a certain weight (required capacity). A selection is feasible if the total weight of the selected items does not exceed a maximum weight (available capacity). Figure 1 shows the formal model of a simple knapsack problem.

1 Gallo/Hammer/Simeone (1980)

2 Eilon (1987), 489.

n number of potential production orders

M_i contribution margin of production order i

u_i binary variable,

$u_i = 1$, if production order i is selected

$u_i = 0$, if production order i is not selected

c_i required capacity for production order i

C available capacity

objective function:

$$Max \sum_{i=1}^{n} u_i * M_i$$

subject to the constraints:

$$\sum_{i=1}^{n} c_i * u_i \leq C$$

$$u_i \in \{0, 1\} \quad \forall i = 1,...,n$$

Figure 1: Formal model of a simple knapsack problem

This formal model of a simple knapsack problem is not suitable for solving the production order selection problem because of two reasons:

1. There is only a single capacity constraint.
2. Synergies between production orders are not considered.

Figure 2 outlines a model of an extended knapsack problem. This extended model allows considering several types of capacity, because for each type of capacity a constraint is modelled. Moreover, the objective function is extended by the introduction of synergies $s_{i.j}$ between two production orders i and j. These synergies are defined as cost savings or as sales increases[3]. It is assumed that $s_{i.j}$ is equal to $s_{j.i}$ and that synergies between two production orders i and j are not considered twice. To avoid double accounting of the synergy between two production orders, only pairs of orders (i,j) with $j>i$ are considered. Meaningless syner-

[3] Martin/Eisenhardt (2001), H2

gies $s_{i.j}$ with *i=j* are not relevant, because synergies can only be realized between two different production orders. Furthermore, like in bi-linear ("quadratic") assignment models a second binary variable u_j is introduced. Thereby it is ensured that a synergy $s_{i.j}$ is only considered in the objective function value, if both (and different) production orders *i* and *j* are selected for execution.

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n number of potential production orders

M_i contribution margin of production order *i*

u_i binary variable,

$u_i = 1$, if production order *i* is selected

$u_i = 0$, if production order *i* is <u>not</u> selected

u_j binary variable,

$u_j = 1$, if production order *j* is selected

$u_j = 0$, if production order *j* is <u>not</u> selected

$s_{i.j}$ synergy, if production order *i* and production order *j* are selected

$c_{i.k}$ required capacity of type k for production order *i*

C_k available capacity of type *k*

K number of types of capacities

objective function:

$$Max \sum_{i=1}^{n} u_i * M_i + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j>1}}^{n} s_{i.j} * u_i * u_j$$

subject to the constraints:

$$\sum_{i=1}^{n} c_{i.k} * u_i \leq C_k \quad \forall k = 1,\ldots,K$$

$$u_i \in \{0, 1\} \quad \forall i = 1,\ldots,n$$

$$u_j \in \{0, 1\} \quad \forall j = 1,\ldots,n$$

Figure 2: Formal model of an extended knapsack problem

In practice an enterprise might be forced to execute a production order *i*, although the production order has a low contribution margin and leads to low synergies. This might be the case if, for example, a production order is placed by a key customer. This case can be taken into consideration in both models illustrated in figure 1 and 2. If the production order *i* has to be executed, a constant value of 1 is assigned to the binary variable u_i of production order *i*.

3 A numerical example

The contribution margins M_i and the capacity requirements $c_{i.k}$ for five fictitious production orders are given in Figure 3. If a production order has been selected for execution, the capacity requirements for each type of capacity with $c_{i.k} > 0$ have to be met by the available capacity.

production order	**contribution margin** $[M_i]$	**capacity requirements for each type of capacity** $[c_{i.k}]$				
		1	2	3	4	5
1	2,150	25	20	15	10	5
2	2,800	15	50	50	5	10
3	2,200	15	20	20	10	5
4	1,900	0	20	40	10	10
5	2,250	20	10	20	20	5
	total capacity requirements	75	120	145	55	35

Figure 3: contribution margins and capacity requirements

In Figure 4 the available capacity C_k for each type of capacity is given. The total capacity requirements of all five production orders (see bottom row in Figure 3) exceed the available capacity for all types of capacity. Therefore, a selection has to be made from the production orders.

type of capacity	**available capacity** [C_k]
1	40
2	60
3	80
4	40
5	20

Figure 4: available capacity for each type of capacity

In Figure 5 the synergies $s_{i.j}$ which can be realized by selecting two production orders *i* and *j* are given.

production orders	1	2	3	4	5
1	0	300	200	0	50
2	0	0	450	370	50
3	0	0	0	0	30
4	0	0	0	0	50
5	0	0	0	0	0

Figure 5: synergies [$s_{i.j}$] between production orders

The solution with a maximum objective function value of 6,450 is obtained by selecting the production orders 1, 3, and 4 for execution. This objective function value is the sum of the contribution margins of the productions orders (6,250) and the synergies between the selected production orders (200).

If, for example, production order 2 has to be executed for a regular customer, a constant value of 1 is assigned to the binary variable u_2. This is a modification of the example which leads to a maximum objective function value of 5,100. This objective function value is obtained by selecting production order 5 for execution in addition to the enforced execution of production order 2.

4 Concluding remarks

The models presented in Figure 1 and 2 can be solved using a software package for solving optimisation problems such as Lingo[4].

In practice the amount of work involved in estimating the synergies between the production orders and possible problems to anticipate synergies may impede the application of the extended model. Also the capacity requirements planning for production orders which are not selected for execution causes work effort. But the increased effort caused by the application of the extended model can be justified by the selection of production orders that possibly yields a higher total margin.

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