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Extended Version of EATWOS concerning Satisficing Levels for Input Quantities

Malte L. Peters, Stephan Zelewski and Adina Silvia Bruns

Abstract

The Efficiency Analysis Technique With Output Satisficing (EATWOS) offers in contrast to the Data Envelopment Analysis (DEA) the opportunity to consider satisficing levels for output quantities. In the article at hand, EATWOS is extended to consider satisficing levels for inputs and thus the Efficiency Analysis Technique With Input and Output Satisficing (EATWIOS) is the result. A satisficing level for an input "average inventory on hand", for example, can recommend itself, if lower deviations to an average inventory level are not judged positively since the availability of a product should be ensured in order to prevent out-of-stock situations. A numerical example for the application of EATWIOS is presented in the context of measuring the efficiency of supply chains.

Keywords:

EATWOS, EATWIOS, efficiency analysis, satisficing levels for inputs and outputs, supply chain efficiency

1 Introduction

1.1 Data Envelopment Analysis and its limitations

Over the past several years, efficiency analysis has received a lot of attention in the logistics management literature (Li & O'Brien, 1999; Zokaei & Simons, 2006).

In particular, the Data Envelopment Analysis (DEA) (Cantner, Krüger & Hanusch, 2007, pp. 68; Charnes, Cooper & Rhodes, 1978; Cooper, Seiford & Tone, 2006, 2007; Dyckhoff & Ahn, 2010; Thanassoulis, 2003) has been widely used in the field of logistics (Barros & Athanassiou, 2004; Chang & Chiu, 2010; Chen, Liang & Yang, 2006; Reiner & Hofmann, 2006; Rios & Maçada, 2006). DEA is a technique to analyze the efficiency of Decision Making Units (DMUs). In efficiency analyses DMUs are described by a combination of input quantities and output quantities. DMUs, for example, can be logistic companies, retail stores as well as supply chains. The major advantage of DEA over other efficiency analysis techniques is that for each DMU the best possible efficiency score is obtained by choosing optimal DMU specific importance weights for inputs and outputs endogenously in the respective DEA model (e.g. Thanassoulis, 2003, pp. 75). However, the DEA suffers from three limitations.

Firstly, knowledge of linear programming is required for an understanding of DEA models. This makes it difficult to communicate the results of an efficiency analysis to numerous logistic practitioners who do not have such mathematical knowledge, especially the fact that the importance weights for inputs and outputs are determined endogenously in the model complicates the task of gaining logistic practitioners' acceptance for the results of DEA applications (Peters & Zelewski, 2010, p. 224).

Secondly, the DEA has a low discriminating power in the case of a small number of DMUs. The more inputs and outputs are included the lower is the discriminating power of DEA (Dyson et al., 2001, p. 248). Certainly, several DEA models with constraints on the importance weights for inputs and outputs have been developed (e.g. Cooper, Seiford & Tone, 2006, pp. 165; Thanassoulis, 2003, pp. 202). These DEA models have a higher discriminating power due to the additional constraints (e.g. Peters, 2008, pp. 723), but they are mathematically more sophisticated than basic DEA models, and this makes it more difficult to communicate the results of an efficiency analysis to logistic practitioners without appropriate mathematical background.

For basic DEA models, there are some rules of thumb for the relation between the number J of outputs and the number K of inputs, on the one hand, and the number of DMUs, on the other hand, proposed in the DEA literature (Cooper, Seiford & Tone, 2006, p. 272, 2007, p. 116 and p. 284; Dyson et al., 2001, p 248). All these rules of thumb require the number of DMUs to exceed the sum of the number of inputs and the number of outputs (Peters, 2008, p. 723). Dyson et al. (2001, p. 248) suggest that the number of DMUs needs to be equal to or greater than $\boxed{max\{J*K;3*(J+K)\}}$, while Cooper, Seiford & Tone (2006, p. 272, 2007, p. 116 and p. 284) recommend that the number of DMUs needs to be equal to or greater than $\boxed{}$. Thus, in case of two

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$2*J*K$

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$max\{J*K;3*(J+K)\}$

inputs and two outputs according to the rule of thumb by Dyson et al. (2001, p. 248) at least 8 DMUs are required and the rule of thumb by Cooper, Seiford & Tone (2006, p. 272, 2007, p. 116 and p. 284) suggests to include at least 12 DMUs into the efficiency analysis. However, logistic practitioners will face many efficiency analysis problems with more inputs and outputs as well as less DMUs.

Thirdly, Simon's concept of satisficing (e.g. Simon 1979, pp. 495) which is often regarded as a core element of bounded rationality has not been considered in the basic DEA models (e.g. Charnes, Cooper & Rhodes, 1978, pp. 430; Cooper, Seiford & Tone, 2006, pp. 21; Thanassoulis, 2003, pp. 65). Certainly the concept of satisficing has been incorporated into stochastic DEA models to consider aspiration levels determined as efficiency scores which are to be reached (Cooper, Huang & Li, 1996, pp. 283; Cooper, Seiford & Tone, 2006, pp. 286), but to the knowledge of the authors up to now there are no DEA models available that offer the opportunity to consider satisficing levels for inputs or outputs.

1.2 Efficiency analysis techniques as practice-oriented alternatives to the Data Envelopment Analysis

The Efficiency Analysis Technique With Output Satisficing (EATWOS) (e.g. Peters & Zelewski, 2006, pp. 3; Peters & Zelewski, 2012, pp. 3) and its extended version presented in this paper do not suffer from the three aforementioned limitations. Certainly there are several other techniques – like the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (e.g. Hwang & Yoon, 1981, pp. 128) or a simple variant of the Operational Competitiveness Rating (OCRA) (e.g. Parkan & Wu, 2000, pp. 499; Peters & Zelewski, 2010) – that do not have the first two mentioned limitations, but the third limitation can only be overcome by using EATWOS and its extended version presented in this paper.

In EATWOS the concept of satisficing is integrated into efficiency analysis by offering the opportunity to define a satisficing level for each output. If a satisficing level is defined for an output, a quantity of this output meeting the satisficing level is judged to be just as good as a quantity of this output exceeding the satisficing level. The determination of a satisficing level for an output could recommend itself, if an output cannot be utilized directly and cannot be stored or can only be stored at unjustifiable high costs. A current example in Germany is wind energy that is often produced intermittently in an amount that cannot be added to the electricity grid and that cannot be consumed immediately, while energy storage capacity is not sufficiently available.

In the paper at hand, the extension of EATWOS to consider satisficing levels for inputs is presented, so that the Efficiency Analysis Technique With Input and Output Satisficing (EATWIOS) is the result. If a satisficing level for an input is defined, an input quantity being equal to this satisficing level is judged to be as good as an input quantity below this satisficing level. For example, if a clause that a minimum number of employees must be scheduled to work in each shift is included in a collective labor agreement, a satisficing level for the input "employees per shift" should be set to this minimum number in an efficiency analysis. Then, a shift with a number of employees under the predefined minimum does not receive a higher efficiency score in an EATWIOS application with consideration of satisficing levels.

This paper is organized as follows: In section 2, some basic steps of EATWOS and its extended version EATWIOS are described. This section is based on Peters & Zelewski, 2006, 2012. The reader is referred to these publications for a complete description of EATWOS and EATWIOS. Section 3 elucidates the theoretical descriptions of EATWOS and EATWIOS by presenting a practical example in the logistics area. Section 4 concludes the paper with some remarks.

2 EATWOS and its extension to consider satisficing levels for inputs

2.1 Input data preparation

At first, the decision maker has to make up his mind about the DMUs i with $i = 1,...,I$ which should be incorporated in the efficiency analysis. Similarly to other efficiency analysis techniques, it has to be decided which inputs and which outputs should be considered. Then, EATWOS as well as EATWIOS require the decision maker to establish all output quantities y_{ij} as well as all input quantities x_{ik} for all considered DMUs i. The quantities $y_{ij} \in R_{\geq 0}$ of all outputs j $(\forall j = 1,..., J)$ of all DMUs i $(\forall i = 1,..., I)$ have to be entered into the output matrix $\underline{Y}$, while the quantities $x_{ik} \in \Re_{\geq 0}$ of all inputs $(\forall k = 1,..., K)$ of all DMUs have to be inserted into the input matrix $\underline{X}$.

EATWOS, as well as its extension EATWIOS, requires that inputs and outputs are cardinal measures. If ordinal inputs and/or ordinal outputs are to be considered, the input and output quantities have to be established by employing a simple scoring technique or more elaborated techniques like the Analytic Hierarchy Process (AHP). EATWOS offers the opportunity to consider a satisficing level for each output. However, the consideration of satisficing levels for outputs is not part of this paper. EATWIOS additionally provides the possibility to determine a satisficing level $\boxed{SL_k \in \Re_{\geq 0}}$ for each input k. EATWOS and EATWIOS require the exogenous

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assessment of the relative importance weights v_j of the outputs and the relative importance weights w_k of the inputs. These importance weights can be determined by employing a scoring technique.

2.2 Application of EATWIOS without consideration of satisficing levels

The first step is to apply EATWIOS without consideration of satisficing levels. If information about potential efficiency improvements is not desired, this step could be omitted.

Firstly, the output quantities y_{ij} are normalized. The normalization of the output quantities takes place as in TOPSIS (Hwang & Yoon, 1981, p. 131).

$$\left(\exists i = 1,\dots,I \ \exists j = 1,\dots,J:\ y_{ij} \neq 0\right) \ \rightarrow \ \forall i = 1,\dots,I \ \forall j = 1,\dots,J:\ r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{h=1}^{I} y_{hj}^2}}$$

$$\left(\forall i = 1,\dots,I \ \forall j = 1,\dots,J:\ y_{ij} = 0\right) \ \rightarrow \ \forall i = 1,\dots,I \ \forall j = 1,\dots,J:\ r_{ij} = 0$$

The result of the normalization process is the normalized output matrix $\underline{R}$. The maximum normalized output quantity r_j^* is determined for each output j on basis of the column vectors of the normalized output matrix $\underline{R}$.

EATWIOS recurs to distance measures similar to known efficiency analysis techniques like DEA and OCRA. The distance measures op_{ij} for the outputs can be calculated on the basis of the matrix $\underline{R}$ and the maximum normalized output quantities r_j^*.

$$op_{ij} = 1 - \left(r_j^* - r_{ij}\right) \qquad \forall i = 1,\ldots,I \quad \forall j = 1,\ldots,J$$

The normalization of the input quantities x_{ik} takes place analogously to the output quantities.

$$\left(\exists i = 1,\ldots,I \;\; \exists k = 1,\ldots,K:\; x_{ik} \neq 0\right) \;\; \rightarrow \;\; \forall i = 1,\ldots,I \; \forall k = 1,\ldots,K:\; s_{ik} = \frac{x_{ik}}{\sqrt{\sum_{h=1}^{I} x_{hk}^2}}$$

$$\left(\forall i = 1,\ldots,I \; \forall k = 1,\ldots,K:\; x_{ik} = 0\right) \;\; \rightarrow \;\; \forall i = 1,\ldots,I \; \forall k = 1,\ldots,K:\; s_{ik} = 0$$

Subsequently, the minimum normalized input quantity s_k^* is determined for each input k on basis of the column vectors of the normalized input matrix $\underline{S}$.

Then, the input distance measure ip_{ik} can be calculated by adding the respective value s_{ik} from the matrix $\underline{S}$ to 1 and subtracting the minimum normalized input quantity s_k^*.

$$ip_{ik} = 1 + s_{ik} - s_k^* \qquad \forall i = 1,\ldots,I \quad \forall k = 1,\ldots,K$$

The input distance measures (input scores) and the output distance measures (output scores) are utilized to calculate an efficiency score E_i for each DMU:

$$E_i = \frac{\sum_{j=1}^{J} v_j * op_{ij}}{\sum_{k=1}^{K} w_k * ip_{ik}} \qquad \forall i = 1,\ldots,I$$

This efficiency score has the classical form in which the output scores are included in the numerator and the input scores are considered in the denominator. This kind of efficiency scores has been widely used in the literature. For example, the DEA utilizes a similar efficiency score (e.g. Charnes, Cooper & Rhodes, 1978, p. 430; Peters, 2008, p. 731; Rios & Maçada, 2006, p. 334).

A low efficiency score E_i of a DMU i indicates a low efficiency relative to the other DMUs, while a high efficiency score E_i means a high efficiency. Thus, based on these efficiency scores, a rank order R of the efficiency of the DMUs can be derived by sorting the efficiency scores from high to low.

2.3 Application of EATWIOS with consideration of satisficing levels for inputs

In this step a satisficing level SL_k is considered for at least one of the inputs k with $k \in \{1,...,K\}$. The inputs without satisficing levels are treated as described in the previous section. The idea of input satisficing is incorporated into EATWOS by embodying the following logical rule: If the input quantity x_{ik} of a DMU i falls short of a certain satisficing level SL_k, then the DMU receives the same input score as a DMU with an input quantity equal to the satisficing level SL_k. In order to model this rule, the following seven constraints are applied for all inputs with a satisficing level:

[1a'] $\left(\dfrac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \leq 1 \qquad \forall i = 1,...,I$

[1b'] $\left(\dfrac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) * q_{2.ik} \geq 0 \qquad \forall i = 1,...,I$

[2'] $q_{1.ik}, q_{2.ik} \in \{0;1\} \qquad \forall i = 1,...,I$

[3'] $q_{1.ik} + q_{2.ik} = 1 \qquad \forall i = 1,...,I$

[4'] $c_{ik} = \dfrac{x_{ik}}{SL_k} * q_{2.ik} + 1 * q_{1.ik} \qquad \forall i = 1,...,I$

[5'] $x_{ik} > 0 \qquad \forall i = 1,...,I$

[6'] $SL_k \geq \dfrac{x_{ik}}{x_{ik} + 1} \qquad \forall i = 1,...,I$

Since not for each input k a satisficing level SL_k has to be specified, it follows for the modified input quantities c_{ik}:

$$\forall i = 1,...,I \quad \forall k = 1,...,K: \quad c_{ik} = \begin{cases} \dfrac{x_{ik}}{SL_k} * q_{2.ik} + 1 * q_{1.ik}\,; & \text{if a satisficing level } SL_k \text{ is defined} \\ s_{ik}\,; & \text{otherwise} \end{cases}$$

The constraints [1a'] and [1b'] serve to restrict the possible values of the logical variables $q_{1.ik}$ and $q_{2.ik}$. Constraint [2'] defines the logical variables as binary variables. Constraint [3'] ensures in connection with constraint [2'] that only one of the logical variables can take the value one, while the other one takes the value zero. The possible values of the logical variables in constraint [4'] are determined by the constraints [1a'], [1b'], [2'], [3'], [5'] und [6']. The positivity constraint [5'] ensures that a division by zero is prevented. Constraint [6'] is required, since constraint [1a'] would not be satisfiable, if the satisficing level SL_k in the denominator of the fraction term would be smaller than the numerator $(x_{ik} - SL_k) : x_{ik}$ of the fraction term.

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SL_k

Depending on the values of an input quantity x_{ik} and an accompanying satisficing level the following three value combinations are possible:

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a) The input quantity x_{ik} takes a value between zero and the satisficing level SL_k $(0 < x_{ik} < SL_k)$:

$$[1a'] \quad \left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \leq 1 \qquad \| \quad 0 < x_{ik} < SL_k$$

$$\Rightarrow \quad]-\infty;0[+ q_{1.ik} \leq 1 \qquad \| \quad q_{1.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{1.ik} \in \{0;1\}$$

$$[1b'] \quad \left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) * q_{2.ik} \geq 0 \qquad \| \quad 0 < x_{ik} < SL_k$$

$$\Rightarrow \quad]-\infty;0[* q_{2.ik} \geq 0 \qquad \| \quad q_{2.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{2.ik} = 0$$

$$q_{2.ik} = 0 \quad \wedge \quad [2'] \quad \wedge \quad [3'] \quad \Rightarrow \quad q_{1.ik} = 1$$

$$[4'] \qquad c_{ik} = \frac{x_{ik}}{SL_k} * 0 + 1 * 1 = 1$$

b) The input quantity x_{ik} is equal to the satisficing level SL_k $(x_{ik} = SL_k)$:

$$[1a'] \quad \left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \leq 1 \qquad \| \quad x_{ik} = SL_k$$

$$\Rightarrow \quad 0 + q_{1.ik} \leq 1 \qquad \| \quad q_{1.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{1.ik} \in \{0;1\}$$

$$[1b'] \quad \left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) * q_{2.ik} \geq 0 \qquad \| \quad x_{ik} = SL_k$$

$$\Rightarrow \quad 0 * q_{2.ik} \geq 0 \qquad \| \quad q_{2.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{2.ik} \in \{0;1\}$$

Because of [3'], [1a'] and [1b'] are consistent with two alternative cases yielding to the same value of c_{ik}.

$$[4'] \quad \left.\begin{array}{l} q_{1.ik} = 0 \quad \wedge \quad q_{2.ik} = 1 \\ c_{ik} = \dfrac{SL_k}{SL_k} * 1 + 1 * 0 = 1 \\ \vee \; q_{1.ik} = 1 \quad \wedge \quad q_{2.ik} = 0 \\ c_{ik} = \dfrac{SL_k}{SL_k} * 0 + 1 * 1 = 1 \end{array}\right\} \Rightarrow c_{ik} = 1$$

c) The input quantity x_{ik} is greater than the satisficing level SL_k $(x_{ik} > SL_k)$:

$$[1a'] \quad \left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \leq 1 \qquad \| \quad x_{ik} > SL_k$$

$$\Rightarrow \quad]0;1[+ q_{1.ik} \leq 1 \qquad \| \quad q_{1.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{1.ik} = 0$$

$$q_{1.ik} = 0 \quad \wedge \quad [2'] \wedge \quad [3'] \quad \Rightarrow \quad q_{2.ik} = 1$$

$$[1b'] \quad \left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) * q_{2.ik} \geq 0 \qquad \| \quad x_{ik} > SL_k$$

$$\Rightarrow \quad]0;1[* q_{2.ik} \geq 0 \qquad \| \quad q_{2.ik} = 1$$

$$\Rightarrow \quad]0;1[* 1 \geq 0$$

Errata — ✓Correction

$$[4'] \quad c_{ik} = \frac{x_{ik}}{SL_k} * 1 + 1 * 0 = \frac{x_{ik}}{SL_k}$$

The modified input quantities c_{ik} can take values that are considerably greater than one. This can lead to a not intended implicit stronger weighting of those inputs for which a satisficing level SL_k is determined. The modified input quantities c_{ik} are normalized in order to prevent this weighting effect:

$$[7'] \quad \forall i = 1,\ldots,I \quad \forall k = 1,\ldots,K: \quad c_{ik}^{n} = \begin{cases} \dfrac{c_{ik}}{\sqrt{\sum_{h=1}^{I} c_{hk}^{2}}}; & \text{if a satisficing level } SL_k \text{ is defined} \\ s_{ik}; & \text{otherwise} \end{cases}$$

These normalized modified input quantities c_{ik}^{n} are determined by the constraints [1a'], [1b'], [2'], [3'], [4'], [5'], [6'] and [7'], if a satisficing level SL_k is established for the respective input k. The normalized modified input quantities c_{ik}^{n} are entered into the normalized input matrix $\underline{C}^{n}$. If no satisficing level SL_k is established for an input k, the respective column vector in the matrix $\underline{C}^{n}$ is equal to the column vector in the normalized input matrix $\underline{S}$.

After that the minimum normalized modified input quantity $c_{k}^{n^*}$ is determined for each input k by taking the minimum value of the respective column vector. The minimum normalized input quantity $c_{k}^{n^*}$ serves to calculate the input distance measures under consideration of satisficing levels. These input distance measures $ip_{ik}^{SL_k}$ are calculated for all DMUs i and for all inputs k.

$$ip_{ik}^{SL_k} = 1 + c_{ik}^{n} - c_{k}^{n^*} \qquad \forall i = 1,\ldots,I \quad \forall k = 1,\ldots,K$$

Then, the efficiency scores $E_{i}^{SL_k}$ are calculated for each DMU. These efficiency scores are incorporating the input distance measures $ip_{ik}^{SL_k}$:

$$E_{i}^{SL_k} = \frac{\sum_{j=1}^{J} v_j * op_{ij}}{\sum_{k=1}^{K} w_k * ip_{ik}^{SL_k}} \qquad \forall i = 1,\ldots,I$$

The efficiency scores $E_{i}^{SL_k}$ do incorporate the input distance measures $ip_{ik}^{SL_k}$, so that the satisficing levels SL_k for the inputs are considered. A rank order R^{SL_k} of the efficiency of the DMUs can be obtained once again by sorting the efficiency scores $E_{i}^{SL_k}$ from high to low.

If the rank order R^{SL_k} based on the efficiency scores incorporating the satisficing levels differs from the rank order R, this is an indication that the efficiency of a DMU ranked lower with respect to rank order R than with rank order R^{SL_k} could be improved. Because a higher rank of a DMU in the rank order R^{SL_k} indicates that this DMU may produce higher output quantities with the same input quantities, since the higher rank is due to the consideration of at least one satisficing level SL_k. The DMU (or DMUs) with a lower rank in the rank order R^{SL_k} in comparison to the rank order R may possibly have a favorable ratio of output quantities to input quantities than a DMU that has "overtaken" this DMU (or these DMUs) in rank order R^{SL_k}. For a DMU that has overtaken at least one other DMU in the rank order R^{SL_k} can be examined whether the production process can be improved in order to achieve a higher efficiency score $E_i^{SL_k}$.

3 Numerical Example: Measuring Supply Chain Efficiency

In the following numerical example EATWIOS is applied to measure the efficiency of three supply chains with the inputs and outputs shown in Table 1. The period of analysis considered is one month. For input 2 "average inventory on hand" a satisficing level is determined, since empty shelf spaces do not have a positive effect on customers and thus out-of-stock situations should be prevented for guaranteeing a minimum service level of the supply chain considered. The satisficing level for input 2 is set to 10 items of material. Output 2 "perfect deliveries" is a percentage of the deliveries to the final stage of the respective supply chain. The input quantities and the output quantities as well as the relative importance weights are given in Table 1. The intermediate results are rounded to three digits after each step.

	input 1	input 2	output 1	output 2
	total supply chain costs	average inventory on hand	sales	perfect deliveries
relative importance weights	0.7	0.3	0.8	0.2
supply chain 1	3,180	6.625	14,151	99.6 %
supply chain 2	3,705	10.625	21,255	98.7 %
supply chain 3	4,914	17.167	24,948	99.3 %

Tab. 1: Input quantities, output quantities and relative importance weights

Firstly, the output quantities as well as the input quantities have to be normalized and the maximum normalized output quantities as well as the minimum normalized input quantities have to be determined. The results are given in Table 2.

	input 1	input 2	output 1	output 2
supply chain 1	0.459	0.312	0.396	0.580
supply chain 2	0.535	0.500	0.595	0.574
supply chain 3	0.709	0.808	0.699	0.578
	minimum normalized input quantities		maximum normalized output quantities	
	0.459	0.312	0.699	0.580

Tab. 2: Normalized input quantities and normalized output quantities

After that, the input distance measures and the output distance measures are calculated and given in Table 3.

	input distance measures		output distance measures	
	input 1	input 2	output 1	output 2
supply chain 1	1.000	1.000	0.697	1.000
supply chain 2	1.076	1.188	0.896	0.994
supply chain 3	1.250	1.496	1.000	0.998

Tab. 3: Distance measures without consideration of satisficing levels

Subsequently, the efficiency scores have to be calculated:

Errata Correction E_i

$$E_1 = \frac{0.800 * 0.697 + 0.200 * 1.000}{0.700 * 1.000 + 0.300 * 1.000} \approx 0.758 \qquad E_2 \approx 0.826 \quad E_3 \approx 0.755$$

The rank order R is as follows:

1.) supply chain 2, 2.) supply chain 1, 3.) supply chain 3.

Then, the eight constraints, presented in section 2.3, have to be applied to input 2, since a satisficing level of 10 items is defined for this input, in order to determine the modified input quantities. These modified input quantities are normalized. The values are given in Table 4.

	modified input quantities	normalized modified input quantities
supply chain 1	1.000	0.444
supply chain 2	1.063	0.472
supply chain 3	1.717	0.762

Tab. 4: Modified input quantities for input 2 considering the satisficing level

Table 5 shows the input distance measures for input 2 that are calculated on basis of the normalized modified input quantities from Table 4.

supply chain 1	1.000
supply chain 2	1.028
supply chain 3	1.318

Tab. 5: Input distance measures for input 2 considering the satisficing level

Then, the efficiency scores incorporating the satisficing level for input 2 have to be calculated.
Errata — Correction $E_i^{SL_k}$

$$E_1^{SL_k} = E_1 \approx 0.758 \quad E_2^{SL_k} = \frac{0.800 * 0.896 + 0.200 * 0.994}{0.700 * 1.076 + 0.300 * 1.028} \approx \frac{0.717 + 0.199}{0.753 + 0.308} \approx 0.863$$

$$E_3^{SL_k} \approx 0.787$$

Therefore, the rank order R^{SL_k} incorporating the satisficing level, inverting the ranking positions of supply chain 1 and supply chain 3, is as follows:

1.) supply chain 2, 2.) supply chain 3, 3.) supply chain 1.

There are possibly ways to improve the efficiency of supply chain 3, since the rank order R^{SL_k} differs from the rank order and supply chain 3 has with respect to R a
Errata — Correction R
lower efficiency than with respect to R^{SL_k}. Since supply chain 3 benefits from the satisficing level, decreasing the quantity of input 2 can be a starting point for improving the efficiency of supply chain 3 measured in terms of rank order R. However, it should be checked, whether the quantity of input 2 can be decreased without negatively affecting the other quantities.

4 Concluding remarks

It has been demonstrated that EATWIOS is suitable for a number of efficiency analysis problems in the field of logistics. The great advantage of EATWIOS is the opportunity to consider satisficing levels for inputs and for outputs, since the consideration of satisficing levels offers the opportunity to better capture the reality in several cases. Apart from the examples aforementioned in this paper satisficing levels can be defined to ease the pressure on employees. But this is a double edged sword, since hard working employees possibly would regard the consideration of satisficing levels as a penalty. However, satisficing levels are additional parameters that can be misused to manipulate the results of an efficiency analysis.

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Extended version of EATWOS
concerning satisficing levels for input quantities

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Abstract

The Efficiency Analysis Technique With Output Satisficing (EATWOS) offers in contrast to the Data Envelopment Analysis (DEA) the opportunity to consider satisficing levels for output quantities. In the article at hand, EATWOS is extended to consider satisficing levels for inputs and thus the Efficiency Analysis Technique With Input and Output Satisficing (EATWIOS) is the result. A satisficing level for an input “average inventory on hand”, for example, can recommend itself, if lower deviations to an average inventory level are not judged positively since the availability of a product should be ensured in order to prevent out-of-stock situations. A numerical example for the application of EATWIOS is presented in the context of measuring the efficiency of supply chains.

Keywords:

EATWOS, EATWIOS, efficiency analysis, satisficing levels for inputs and outputs, supply chain efficiency

1. Introduction

1.1 Data Envelopment Analysis and its limitations

Over the past several years, efficiency analysis has received a lot of attention in the logistics management literature (Li & O’Brien, 1999; Zokaei & Simons, 2006).

In particular, the Data Envelopment Analysis (DEA) (Cantner, Krüger & Hanusch, 2007, pp. 68; Charnes, Cooper & Rhodes, 1978; Cooper, Seiford & Tone, 2006, 2007; Dyckhoff & Ahn, 2010; Thanassoulis, 2003) has been widely used in the field of logistics (Barros & Athanassiou, 2004; Chang & Chiu, 2010; Chen, Liang & Yang, 2006; Reiner & Hofmann, 2006; Rios & Maçada, 2006). DEA is a technique to analyze the efficiency of Decision Making Units (DMUs). In efficiency analyses DMUs are described by a combination of input quantities and output quantities. DMUs, for example, can be logistic companies, retail stores as well as supply chains. The major advantage of DEA over other efficiency analysis techniques is that for each DMU the best possible efficiency score is obtained by choosing optimal DMU specific importance weights for inputs and outputs endogenously in the respective DEA model (e.g. Thanassoulis, 2003, pp. 75). However, the DEA suffers from three limitations.

Firstly, knowledge of linear programming is required for an understanding of DEA models. This makes it difficult to communicate the results of an efficiency analysis to numerous logistic practitioners who do not have such mathematical knowledge, especially the fact that the importance weights for inputs and outputs are determined endogenously in the model complicates the task of gaining logistic practitioners' acceptance for the results of DEA applications (Peters & Zelewski, 2010, p. 224).

Secondly, the DEA has a low discriminating power in the case of a small number of DMUs. The more inputs and outputs are included the lower is the discriminating power of DEA (Dyson *et al.*, 2001, p. 248). Certainly, several DEA models with constraints on the importance weights for inputs and outputs have been developed (e.g. Cooper, Seiford & Tone, 2006, pp. 165; Thanassoulis, 2003, pp. 202). These DEA models have a higher discriminating power due to the additional constraints (e.g. Peters, 2008, pp. 723), but they are mathematically more sophisticated than basic DEA models, and this makes it more difficult to communicate the results of an efficiency analysis to logistic practitioners without appropriate mathematical background.

For basic DEA models, there are some rules of thumb for the relation between the number J of outputs and the number K of inputs, on the one hand, and the number of DMUs, on the other hand, proposed in the DEA literature (Cooper, Seiford & Tone, 2006, p. 272, 2007, p. 116 and p. 284; Dyson *et al.*, 2001, p 248). All these rules of thumb require the number of DMUs to exceed the sum of the number of inputs and the number of outputs (Peters, 2008, p. 723). Dyson *et al.* (2001, p. 248) suggest that the number of DMUs needs to be equal to or greater than $2 * J * K$, while Cooper, Seiford & Tone (2006, p. 272, 2007, p. 116 and p. 284) recommend that the number of DMUs needs to be equal to or greater than $max\left\{ J * K; 3 * (J + K) \right\}$. Thus, in case of two inputs and two outputs according to the rule of thumb by Dyson *et al.* (2001, p. 248) at least 8 DMUs are required and the rule of thumb by Cooper, Seiford & Tone (2006, p. 272, 2007, p. 116 and p. 284) suggests to include at least 12 DMUs into the efficiency analysis. However, logistic practitioners will face many efficiency analysis problems with more inputs and outputs as well as less DMUs.

Thirdly, Simon's concept of satisficing (e.g. Simon 1979, pp. 495) which is often regarded as a core element of bounded rationality has not been considered in the basic DEA models (e.g. Charnes, Cooper & Rhodes, 1978, pp. 430; Cooper, Seiford & Tone, 2006, pp. 21; Thanassoulis, 2003, pp. 65). Certainly the concept of satisficing has been incorporated into stochastic DEA models to consider aspiration levels determined as efficiency scores which are to be reached (Cooper, Huang & Li, 1996, pp. 283; Cooper, Seiford & Tone, 2006, pp. 286), but to the knowledge of the authors up to now there are no DEA models available that offer the opportunity to consider satisficing levels for inputs or outputs.

1.2 Efficiency analysis techniques as practice-oriented alternatives to the Data Envelopment Analysis

The Efficiency Analysis Technique With Output Satisficing (EATWOS) (e.g. Peters & Zelewski, 2006, pp. 3; Peters & Zelewski, 2012, pp. 3) and its extended version presented in this paper do not suffer from the three aforementioned limitations. Certainly there are several other techniques – like the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (e.g. Hwang & Yoon, 1981, pp. 128) or a simple variant of the Operational Competitiveness Rating (OCRA) (e.g. Parkan & Wu, 2000, pp. 499; Peters & Zelewski, 2010) – that do not have the first two mentioned limitations, but the third limitation can only be overcome by using EATWOS and its extended version presented in this paper.

In EATWOS the concept of satisficing is integrated into efficiency analysis by offering the opportunity to define a satisficing level for each output. If a satisficing level is defined for an output, a quantity of this output meeting the satisficing level is judged to be just as good as a quantity of this output exceeding the satisficing level. The determination of a satisficing level for an output could recommend itself, if an output cannot be utilized directly and cannot be stored or can only be stored at unjustifiable high costs. A current example in Germany is wind energy that is often produced intermittently in an amount that cannot be added to the electricity grid and that cannot be consumed immediately, while energy storage capacity is not sufficiently available.

In the paper at hand, the extension of EATWOS to consider satisficing levels for inputs is presented, so that the Efficiency Analysis Technique With Input and Output

Satisficing (EATWIOS) is the result. If a satisficing level for an input is defined, an input quantity being equal to this satisficing level is judged to be as good as an input quantity below this satisficing level. For example, if a clause that a minimum number of employees must be scheduled to work in each shift is included in a collective labor agreement, a satisficing level for the input "employees per shift" should be set to this minimum number in an efficiency analysis. Then, a shift with a number of employees under the predefined minimum does not receive a higher efficiency score in an EATWIOS application with consideration of satisficing levels.

This paper is organized as follows: In section 2, some basic steps of EATWOS and its extended version EATWIOS are described. This section is based on Peters & Zelewski, 2006, 2012. The reader is referred to these publications for a complete description of EATWOS and EATWIOS. Section 3 elucidates the theoretical descriptions of EATWOS and EATWIOS by presenting a practical example in the logistics area. Section 4 concludes the paper with some remarks.

2. EATWOS and its extension to consider satisficing levels for inputs

2.1 Input data preparation

At first, the decision maker has to make up his mind about the DMUs i with $i = 1,...,I$ which should be incorporated in the efficiency analysis. Similarly to other efficiency analysis techniques, it has to be decided which inputs and which outputs should be considered. Then, EATWOS as well as EATWIOS require the decision maker to establish all output quantities y_{ij} as well as all input quantities x_{ik} for all considered DMUs i. The quantities $y_{ij} \in \mathbb{R}_{\geq 0}$ of all outputs j ($\forall j = 1,...,J$) of all DMUs i ($\forall i = 1,...,I$) have to be entered into the output matrix $\underline{Y}$, while the quantities $x_{ik} \in \mathbb{R}_{\geq 0}$ of all inputs ($\forall k = 1,...,K$) of all DMUs have to be inserted into the input matrix $\underline{X}$.

EATWOS, as well as its extension EATWIOS, requires that inputs and outputs are cardinal measures. If ordinal inputs and/or ordinal outputs are to be considered, the input and output quantities have to be established by employing a simple scoring technique or more elaborated techniques like the Analytic Hierarchy Process (AHP).

EATWOS offers the opportunity to consider a satisficing level for each output. However, the consideration of satisficing levels for outputs is not part of this paper. EATWIOS additionally provides the possibility to determine a satisficing level $SL_k \in \mathbb{R}_{>0}$ for each input k. EATWOS and EATWIOS require the exogenous assessment of the relative importance weights v_j of the outputs and the relative importance weights w_k of the inputs. These importance weights can be determined by employing a scoring technique.

2.2 Application of EATWIOS without consideration of satisficing levels

The first step is to apply EATWIOS without consideration of satisficing levels. If information about potential efficiency improvements is not desired, this step could be omitted.

Firstly, the output quantities y_{ij} are normalized. The normalization of the output quantities takes place as in TOPSIS (Hwang & Yoon, 1981, p. 131).

$$\left(\exists i = 1,\dots,I \;\; \exists j = 1,\dots,J:\; y_{ij} \neq 0\right) \;\; \rightarrow \;\; \forall i = 1,\dots,I \; \forall j = 1,\dots,J:\; r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{h=1}^{I} y_{hj}^2}}$$

$$\left(\forall i = 1,\dots,I \; \forall j = 1,\dots,J:\; y_{ij} = 0\right) \;\; \rightarrow \;\; \forall i = 1,\dots,I \; \forall j = 1,\dots,J:\; r_{ij} = 0$$

The result of the normalization process is the normalized output matrix $\underline{R}$. The maximum normalized output quantity r_j^* is determined for each output j on basis of the column vectors of the normalized output matrix $\underline{R}$.

EATWIOS recurs to distance measures similar to known efficiency analysis techniques like DEA and OCRA. The distance measures op_{ij} for the outputs can be calculated on the basis of the matrix $\underline{R}$ and the maximum normalized output quantities r_j^*.

$$op_{ij} = 1 - \left(r_j^* - r_{ij}\right) \qquad \forall i = 1,\dots,I \quad \forall j = 1,\dots,J$$

The normalization of the input quantities x_{ik} takes place analogously to the output quantities.

$$\left(\exists i=1,\ldots,I \;\; \exists k=1,\ldots,K:\; x_{ik}\neq 0\right) \;\;\rightarrow\;\; \forall i=1,\ldots,I\;\forall k=1,\ldots,K:\; s_{ik}=\frac{x_{ik}}{\sqrt{\sum_{h=1}^{I}x_{hk}^{2}}}$$

$$\left(\forall i=1,\ldots,I\;\forall k=1,\ldots,K:\; x_{ik}=0\right) \;\;\rightarrow\;\; \forall i=1,\ldots,I\;\forall k=1,\ldots,K:\; s_{ik}=0$$

Subsequently, the minimum normalized input quantity s_k^* is determined for each input k on basis of the column vectors of the normalized input matrix $\underline{S}$.

Then, the input distance measure ip_{ik} can be calculated by adding the respective value s_{ik} from the matrix $\underline{S}$ to 1 and subtracting the minimum normalized input quantity s_k^*.

$$ip_{ik}=1+s_{ik}-s_k^* \qquad \forall i=1,\ldots,I \quad \forall k=1,\ldots,K$$

The input distance measures (input scores) and the output distance measures (output scores) are utilized to calculate an efficiency score E_i for each DMU:

$$E_i=\frac{\sum_{j=1}^{J}v_j * op_{ij}}{\sum_{k=1}^{K}w_k * ip_{ik}} \qquad \forall i=1,\ldots,I$$

This efficiency score has the classical form in which the output scores are included in the numerator and the input scores are considered in the denominator. This kind of efficiency scores has been widely used in the literature. For example, the DEA utilizes a similar efficiency score (e.g. Charnes, Cooper & Rhodes, 1978, p. 430; Peters, 2008, p. 731; Rios & Maçada, 2006, p. 334).

A low efficiency score E_i of a DMU i indicates a low efficiency relative to the other DMUs, while a high efficiency score E_i means a high efficiency. Thus, based on these efficiency scores, a rank order R of the efficiency of the DMUs can be derived by sorting the efficiency scores from high to low.

2.3 Application of EATWIOS with consideration of satisficing levels for inputs

In this step a satisficing level SL_k is considered for at least one of the inputs k with $k\in\{1,\ldots,K\}$. The inputs without satisficing levels are treated as described in the previous section. The idea of input satisficing is incorporated into EATWOS by embodying the following logical rule: If the input quantity x_{ik} of a DMU i falls short of a certain satisficing level SL_k, then the DMU receives the same input score as a DMU with an input quantity equal to the satisficing level SL_k. In order to model this rule, the following seven constraints are applied for all inputs with a satisficing level:

[1a'] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \leq 1 \qquad \forall i = 1,...,I$$

[1b'] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) * q_{2.ik} \geq 0 \qquad \forall i = 1,...,I$$

[2'] $$q_{1.ik}, q_{2.ik} \in \{0;1\} \qquad \forall i = 1,...,I$$

[3'] $$q_{1.ik} + q_{2.ik} = 1 \qquad \forall i = 1,...,I$$

[4'] $$c_{ik} = \frac{x_{ik}}{SL_k} * q_{2.ik} + 1 * q_{1.ik} \qquad \forall i = 1,...,I$$

[5'] $$x_{ik} > 0 \qquad \forall i = 1,...,I$$

[6'] $$SL_k \geq \frac{x_{ik}}{x_{ik} + 1} \qquad \forall i = 1,...,I$$

Since not for each input k a satisficing level SL_k has to be specified, it follows for the modified input quantities c_{ik}:

$$\forall i = 1,...,I \quad \forall k = 1,...,K: \quad c_{ik} = \begin{cases} \frac{x_{ik}}{SL_k} * q_{2.ik} + 1 * q_{1.ik}\ ; & \text{if a satisficing level } SL_k \text{ is defined} \\ s_{ik}\ ; & \text{otherwise} \end{cases}$$

The constraints [1a'] and [1b'] serve to restrict the possible values of the logical variables $q_{1.ik}$ and $q_{2.ik}$. Constraint [2'] defines the logical variables as binary variables. Constraint [3'] ensures in connection with constraint [2'] that only one of the logical variables can take the value one, while the other one takes the value zero.

The possible values of the logical variables in constraint [4’] are determined by the constraints [1a’], [1b’], [2’], [3’], [5’] und [6’]. The positivity constraint [5’] ensures that a division by zero is prevented. Constraint [6’] is required, since constraint [1a’] would not be satisfiable, if the satisficing level SL_k in the denominator of the fraction term would be smaller than the numerator $(x_{ik} - SL_k) : x_{ik}$ of the fraction term.

Depending on the values of an input quantity x_{ik} and an accompanying satisficing level SL_k the following three value combinations are possible:

a) The input quantity x_{ik} takes a value between zero and the satisficing level SL_k ($0 < x_{ik} < SL_k$):

[1a’] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \le 1 \qquad \| \quad 0 < x_{ik} < SL_k$$

$$\Rightarrow \quad]-\infty;0[+ q_{1.ik} \le 1 \qquad \| \quad q_{1.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{1.ik} \in \{0;1\}$$

[1b’] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) * q_{2.ik} \ge 0 \qquad \| \quad 0 < x_{ik} < SL_k$$

$$\Rightarrow \quad]-\infty;0[* q_{2.ik} \ge 0 \qquad \| \quad q_{2.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{2.ik} = 0$$

$$q_{2.ik} = 0 \quad \wedge \quad [2'] \quad \wedge \quad [3'] \quad \Rightarrow \quad q_{1.ik} = 1$$

[4’] $$c_{ik} = \frac{x_{ik}}{SL_k} * 0 + 1 * 1 = 1$$

b) The input quantity x_{ik} is equal to the satisficing level SL_k ($x_{ik} = SL_k$):

[1a’] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k}\right) + q_{1.ik} \le 1 \qquad \| \quad x_{ik} = SL_k$$

$$\Rightarrow \quad 0 + q_{1.ik} \leq 1 \qquad \| \quad q_{1.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{1.ik} \in \{0;1\}$$

[1b'] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k} \right) * q_{2.ik} \geq 0 \qquad \| \quad x_{ik} = SL_k$$

$$\Rightarrow \quad 0 * q_{2.ik} \geq 0 \qquad \| \quad q_{2.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{2.ik} \in \{0;1\}$$

Because of [3'], [1a'] and [1b'] are consistent with two alternative cases yielding to the same value of c_{ik}.

[4'] $$\left. \begin{array}{l} q_{1.ik} = 0 \quad \wedge \quad q_{2.ik} = 1 \\ c_{ik} = \frac{SL_k}{SL_k} * 1 + 1 * 0 = 1 \\ \vee \; q_{1.ik} = 1 \quad \wedge \quad q_{2.ik} = 0 \\ c_{ik} = \frac{SL_k}{SL_k} * 0 + 1 * 1 = 1 \end{array} \right\} \Rightarrow c_{ik} = 1$$

c) The input quantity x_{ik} is greater than the satisficing level SL_k ($x_{ik} > SL_k$):

[1a'] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k} \right) + q_{1.ik} \leq 1 \qquad \| \quad x_{ik} > SL_k$$

$$\Rightarrow \quad]0;1[+ q_{1.ik} \leq 1 \qquad \| \quad q_{1.ik} \in \{0;1\} \qquad \text{because of [2']}$$

$$\Rightarrow \quad q_{1.ik} = 0$$

$$q_{1.ik} = 0 \quad \wedge \quad [2'] \wedge \quad [3'] \quad \Rightarrow \quad q_{2.ik} = 1$$

[1b'] $$\left(\frac{(x_{ik} - SL_k) : x_{ik}}{SL_k} \right) * q_{2.ik} \geq 0 \qquad \| \quad x_{ik} > SL_k$$

$$\Rightarrow \quad]0;1[* q_{2.ik} \geq 0 \qquad \| \quad q_{2.ik} = 1$$

$$\Rightarrow \quad]0;1[* 1 \geq 0 \qquad \checkmark$$

[4’] $$c_{ik} = \frac{x_{ik}}{SL_k} * 1 + 1 * 0 = \frac{x_{ik}}{SL_k}$$

The modified input quantities c_{ik} can take values that are considerably greater than one. This can lead to a not intended implicit stronger weighting of those inputs for which a satisficing level SL_k is determined. The modified input quantities c_{ik} are normalized in order to prevent this weighting effect:

[7’] $$\forall i = 1,...,I \quad \forall k = 1,...,K: \quad c_{ik}^n = \begin{cases} \dfrac{c_{ik}}{\sqrt{\sum_{h=1}^{I} c_{hk}^2}}; & \text{if a satisficing level } SL_k \text{ is defined} \\ s_{ik}; & \text{otherwise} \end{cases}$$

These normalized modified input quantities c_{ik}^n are determined by the constraints [1a’], [1b’], [2’], [3’], [4’], [5’], [6’] and [7’], if a satisficing level SL_k is established for the respective input k. The normalized modified input quantities c_{ik}^n are entered into the normalized input matrix $\underline{C}^n$. If no satisficing level SL_k is established for an input k, the respective column vector in the matrix $\underline{C}^n$ is equal to the column vector in the normalized input matrix $\underline{S}$.

After that the minimum normalized modified input quantity c_k^{n*} is determined for each input k by taking the minimum value of the respective column vector. The minimum normalized input quantity c_k^{n*} serves to calculate the input distance measures under consideration of satisficing levels. These input distance measures $ip_{ik}^{SL_k}$ are calculated for all DMUs i and for all inputs k.

$$ip_{ik}^{SL_k} = 1 + c_{ik}^n - c_k^{n*} \qquad \forall i = 1,...,I \quad \forall k = 1,...,K$$

Then, the efficiency scores $E_i^{SL_k}$ are calculated for each DMU. These efficiency scores are incorporating the input distance measures $ip_{ik}^{SL_k}$:

$$E_i^{SL_k} = \frac{\sum_{j=1}^{J} v_j * op_{ij}}{\sum_{k=1}^{K} w_k * ip_{ik}^{SL_k}} \qquad \forall i = 1,...,I$$

The efficiency scores $E_i^{SL_k}$ do incorporate the input distance measures $ip_{ik}^{SL_k}$, so that the satisficing levels SL_k for the inputs are considered. A rank order R^{SL_k} of the efficiency of the DMUs can be obtained once again by sorting the efficiency scores $E_i^{SL_k}$ from high to low.

If the rank order R^{SL_k} based on the efficiency scores incorporating the satisficing levels differs from the rank order R, this is an indication that the efficiency of a DMU ranked lower with respect to rank order R than with rank order R^{SL_k} could be improved. Because a higher rank of a DMU in the rank order R^{SL_k} indicates that this DMU may produce higher output quantities with the same input quantities, since the higher rank is due to the consideration of at least one satisficing level SL_k. The DMU (or DMUs) with a lower rank in the rank order R^{SL_k} in comparison to the rank order R may possibly have a favorable ratio of output quantities to input quantities than a DMU that has “overtaken” this DMU (or these DMUs) in rank order R^{SL_k}. For a DMU that has overtaken at least one other DMU in the rank order R^{SL_k} can be examined whether the production process can be improved in order to achieve a higher efficiency score $E_i^{SL_k}$.

3. Numerical Example: Measuring Supply Chain Efficiency

In the following numerical example EATWIOS is applied to measure the efficiency of three supply chains with the inputs and outputs shown in Table 1. The period of analysis considered is one month. For input 2 “average inventory on hand” a satisficing level is determined, since empty shelf spaces do not have a positive effect on customers and thus out-of-stock situations should be prevented for guaranteeing a minimum service level of the supply chain considered. The satisficing level for input 2 is set to 10 items of material. Output 2 “perfect deliveries” is a percentage of the deliveries to the final stage of the respective supply chain. The input quantities and the output quantities as well as the relative importance weights are given in Table 1. The intermediate results are rounded to three digits after each step.

	input 1	input 2	output 1	output 2
	total supply chain costs	average inventory on hand	sales	perfect deliveries
relative importance weights	0.7	0.3	0.8	0.2
supply chain 1	3,180	6.625	14,151	99.6 %
supply chain 2	3,705	10.625	21,255	98.7 %
supply chain 3	4,914	17.167	24,948	99.3 %

Table 1: Input quantities, output quantities and relative importance weights

Firstly, the output quantities as well as the input quantities have to be normalized and the maximum normalized output quantities as well as the minimum normalized input quantities have to be determined. The results are given in Table 2.

	input 1	input 2	output 1	output 2
supply chain 1	0.459	0.312	0.396	0.580
supply chain 2	0.535	0.500	0.595	0.574
supply chain 3	0.709	0.808	0.699	0.578
	minimum normalized input quantities		maximum normalized output quantities	
	0.459	0.312	0.699	0.580

Table 2: Normalized input quantities and normalized output quantities

After that, the input distance measures and the output distance measures are calculated and given in Table 3.

	input distance measures		output distance measures	
	input 1	input 2	output 1	output 2
supply chain 1	1.000	1.000	0.697	1.000
supply chain 2	1.076	1.188	0.896	0.994
supply chain 3	1.250	1.496	1.000	0.998

Table 3: Distance measures without consideration of satisficing levels

Subsequently, the efficiency scores E_i have to be calculated:

$$E_1 = \frac{0.800 * 0.697 + 0.200 * 1.000}{0.700 * 1.000 + 0.300 * 1.000} \approx 0.758 \qquad E_2 \approx 0.826 \qquad E_3 \approx 0.755$$

The rank order R is as follows:

1.) supply chain 2, 2.) supply chain 1, 3.) supply chain 3.

Then, the eight constraints, presented in section 2.3, have to be applied to input 2, since a satisficing level of 10 items is defined for this input, in order to determine the modified input quantities. These modified input quantities are normalized. The values are given in Table 4.

	modified input quantities	normalized modified input quantities
supply chain 1	1.000	0.444
supply chain 2	1.063	0.472
supply chain 3	1.717	0.762

Table 4: Modified input quantities for input 2 considering the satisficing level

Table 5 shows the input distance measures for input 2 that are calculated on basis of the normalized modified input quantities from Table 4.

supply chain 1	1.000
supply chain 2	1.028
supply chain 3	1.318

Table 5: Input distance measures for input 2 considering the satisficing level

Then, the efficiency scores $E_i^{SL_k}$ incorporating the satisficing level for input 2 have to be calculated.

$$E_1^{SL_k} = E_1 \approx 0.758 \qquad E_2^{SL_k} = \frac{0.800 * 0.896 + 0.200 * 0.994}{0.700 * 1.076 + 0.300 * 1.028} \approx \frac{0.717 + 0.199}{0.753 + 0.308} \approx 0.863$$

$$E_3^{SL_k} \approx 0.787$$

Therefore, the rank order R^{SL_k} incorporating the satisficing level, inverting the ranking positions of supply chain 1 and supply chain 3, is as follows:
1.) supply chain 2, 2.) supply chain 3, 3.) supply chain 1.

There are possibly ways to improve the efficiency of supply chain 3, since the rank order R^{SL_k} differs from the rank order R and supply chain 3 has with respect to R a lower efficiency than with respect to R^{SL_k}. Since supply chain 3 benefits from the satisficing level, decreasing the quantity of input 2 can be a starting point for improving the efficiency of supply chain 3 measured in terms of rank order R. However, it should be checked, whether the quantity of input 2 can be decreased without negatively affecting the other quantities.

4. Concluding remarks

It has been demonstrated that EATWIOS is suitable for a number of efficiency analysis problems in the field of logistics. The great advantage of EATWIOS is the opportunity to consider satisficing levels for inputs and for outputs, since the consideration of satisficing levels offers the opportunity to better capture the reality in several cases. Apart from the examples aforementioned in this paper satisficing levels can be defined to ease the pressure on employees. But this is a double edged sword, since hard working employees possibly would regard the consideration of satisficing levels as a penalty. However, satisficing levels are additional parameters that can be misused to manipulate the results of an efficiency analysis.

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