EFFICIENCY ANALYSIS UNDER CONSIDERATION OF SATISFICING LEVELS FOR OUTPUT QUANTITIES

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Malte L. Peters, University of Duisburg-Essen, Campus Essen
Stephan Zelewski, University of Duisburg-Essen, Campus Essen

Abstract

Normally, efficiency analysis techniques assume that it is desirable to achieve a maximum ratio between the achieved output quantities and the employed input quantities. But Herbert A. Simon received the Nobel Prize in economics partly for the idea of "satisficing". This idea means that individuals tend to search for satisfactory solutions rather than optimal solutions. If Simon's idea is applied to efficiency analysis, it follows that an output quantity meeting a certain satisficing level may be judged to be just as good as an output quantity exceeding this satisficing level. With this in mind, a new efficiency analysis technique is proposed in this paper, which considers satisficing levels for output quantities and identifies efficiency improvement potentials.

1. Introduction

Practitioners often have to analyze the efficiency of so-called Decision Making Units (DMUs) considering multiple inputs and outputs. Usually, in efficiency analysis, it is taken for granted that it is desirable to achieve a maximum ratio between the achieved output quantities and the employed input quantities. A lot of sophisticated techniques like *Data Envelopment Analysis (DEA)* (Charnes, Cooper, and Rhodes 1978; Charnes, Cooper, and Thrall 1991; Cooper, Seiford, and Zhu 2004), *Operational Competitiveness Rating (OCRA)* (Parkan and Wu 1998; Jayanthi, Kocha, and Sinha 1999), and *Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)* (Hwang and Yoon 1981, pp. 128; Janic 2003, pp. 501) can be utilized to analyze the efficiency. All those techniques, however, suffer from a major drawback: they do not incorporate the idea of "satisficing" (e.g. Simon 1979, pp. 495). This idea goes back to the seminal work of Herbert A. Simon who was awarded the Nobel Prize in economics partly for this idea. The core of his idea of "satisficing" is that individuals are searching for satisfactory rather than optimal solutions. If Simon's idea is applied to efficiency analysis, it follows that an output quantity meeting a certain satisficing level may be judged to be just as good as an output quantity exceeding this satisficing level.

In the paper at hand, a new efficiency analysis technique is proposed. This technique integrates the idea of "satisficing" into efficiency analysis by offering the decision maker the opportunity to define a satisficing level for each output. Furthermore, in some cases the proposed efficiency analysis technique is capable of identifying efficiency improvement potentials.

This paper is organized as follows: In section 2, some preliminaries about efficiency are introduced. In section 3, the new type of an efficiency analysis technique named EATWOS (Efficiency Analysis Technique With Output Satisficing) is proposed. Section 4 elucidates the theoretical descriptions of EATWOS by presenting a numerical example. Section 5 concludes the paper with some remarks.

2. Efficiency

As stated above, efficiency is defined normally as the ratio between the achieved output quantities and the employed input quantities. Since usually in efficiency analyses multiple, possibly non-commensurate inputs and outputs are considered, the inputs have to be weighted according to their relative importance and the relative importance of the outputs has to be assessed. The efficiency of a DMU can be evaluated, if a standard of comparison is known. In cases of absolute efficiency, this standard of comparison is given by a production function as the efficient frontier of the technology set. If the technology set is unknown, only a comparison of the DMUs among each other is possible. If, for example, the efficiency of the stores of a retailing group are to be analyzed, there is no production function given, but the efficiency can be analyzed by comparing the stores among themselves. This type of efficiency is called relative efficiency (e.g. Charnes, Cooper, and Rhodes 1978, p. 430). In cases of relative efficiency, the technology set can be partially constructed out of the considered DMUs.

3. The Efficiency Analysis Technique With Output Satisficing (EATWOS)

3.1 Input data preparation

The first step is to decide which inputs and which outputs should be considered. Also, the decision maker has to make up his mind about the DMUs which should be assessed. Then, EATWOS requires the decision maker to establish the output quantities y_{ij} as well as the input quantities x_{ik} for all DMUs. For these reasons, the quantities y_{ij} of all outputs j ($j = 1,...,J$) of all DMUs i ($i = 1,...,I$) have to be entered into the output matrix $\underline{Y}$.

$$\underline{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1j} & \cdots & y_{1J} \\ y_{21} & y_{22} & \cdots & & \cdots & y_{2J} \\ \cdots & \cdots & & & & \cdots \\ y_{i1} & y_{i2} & \cdots & y_{ij} & \cdots & y_{iJ} \\ \cdots & \cdots & & & & \\ y_{I1} & y_{I2} & \cdots & y_{Ij} & \cdots & y_{IJ} \end{bmatrix} \quad \text{with } y_{ij} \in \mathbb{R}_{\geq 0} \quad \forall i = 1,...,I \;\; \forall j = 1,...,J \qquad (1)$$

Each column of this output matrix $\underline{Y}$ corresponds to an output j, and each row corresponds to a DMU i. The input matrix $\underline{X}$ is established in the same way.

$$\underline{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1k} & \cdots & x_{1K} \\ x_{21} & x_{22} & \cdots & & \cdots & x_{2K} \\ \cdots & \cdots & & & & \cdots \\ x_{i1} & x_{i2} & \cdots & x_{ik} & \cdots & x_{iK} \\ \cdots & \cdots & & & & \\ x_{I1} & x_{I2} & \cdots & x_{Ik} & \cdots & x_{IK} \end{bmatrix} \quad \text{with } x_{ik} \in \mathbb{R}_{\geq 0} \quad \forall i = 1,...,I \;\; \forall k = 1,...,K \qquad (2)$$

Analogously to the output matrix, each column of this input matrix $\underline{X}$ corresponds to an input k ($k = 1,...,K$), while each row corresponds to a DMU i. EATWOS requires that inputs and outputs are cardinal measures. If ordinal inputs and/or outputs are to be considered, the input and output quantities have to be established by employing a simple scoring technique or more elaborated techniques like the Analytic Hierarchy Process (AHP) (Saaty 1994, 2004).

Moreover, EATWOS offers the opportunity to consider satisficing levels for outputs. So the decision maker has the possibility to determine a satisficing level SL_j for each output j. If satisficing levels are not to be considered, the decision maker must apply the second step of EATWOS solely as described in the next section. Furthermore, EATWOS requires the exogenous assessment of the relative importance weights v_j of the outputs as well as the relative importance weights w_k of the inputs. These importance weights can be determined as well by employing a scoring technique or the AHP like mentioned above.

In figure 1, all variables needed to apply EATWOS are listed.

model exogenous variables (parameters):

i	DMUs $i = 1,...,I$
j	outputs $j = 1,...,J$
k	inputs $k = 1,...,K$
y_{ij}	quantity of output j of DMU i
x_{ik}	quantity of input k of DMU i
SL_j	satisficing level for output j
v_j	relative importance weight of output j
w_k	relative importance weight of input k

model endogenous variables:

a_{ij}	quantity of output j of DMU i with consideration of the satisficing level SL_j
a_j^*	maximum normalized output quantity with consideration of the satisficing level SL_j
E_i	efficiency score of DMU i without consideration of satisficing levels
E_i^{SL}	efficiency score of DMU i with consideration of satisficing levels
ip_{ik}	distance measure for input k
op_{ij}	distance measure for output j without consideration of satisficing level SL_j
op_{ij}^{SL}	distance measure for output j with consideration of satisficing level SL_j
r_{ij}	normalized quantity of output j of DMU i
r_j^*	maximum normalized output quantity
s_{ik}	normalized quantity of input k of DMU i
s_k^*	minimum normalized input quantity
z_1, z_2	logical variables

Figure 1: Model variables

3.2 Application of EATWOS without consideration of satisficing levels

The second step is to apply EATWOS without consideration of satisficing levels. Thus, satisficing levels are ignored for all outputs. If information about potential efficiency gains is not desired, this step could be omitted.

Firstly, the output quantities y_{ik} are normalized. The normalization of the output quantities takes place as in TOPSIS (Hwang and Yoon 1981, pp. 128).

$$\exists i \quad \exists j \quad y_{ij} \neq 0: \qquad r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{I} y_{ij}^2}} \qquad \forall i = 1,..,I \quad \forall j = 1,...,J \tag{3a}$$

$$\forall i = 1,..,I \quad \forall j = 1,...,J \quad y_{ij} = 0: \qquad r_{ij} = 0 \tag{3b}$$

In the denominator of equation (3a) the term y_{ij} is squared to prevent the denominator from becoming negative and to prevent compensation effects between terms with $y_{ij} < 0$ and $y_{ij} > 0$.

The result of the normalization process is the normalized output matrix $\underline{R}$:

$$\underline{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1j} & \cdots & r_{1J} \\ \cdots & \cdots & & \cdots & & \cdots \\ r_{i1} & r_{i2} & \cdots & r_{ij} & \cdots & r_{iJ} \\ \cdots & \cdots & & \cdots & & \cdots \\ r_{I1} & r_{I2} & \cdots & r_{Ij} & \cdots & r_{IJ} \end{bmatrix} \tag{4}$$

Secondly, the maximum normalized output quantity r_j^* is determined for each output j on basis of the column vectors of $\vec{r}_j$ of the normalized output matrix $\underline{R}$.

$$r_j^* = \max_{\mathrm{i}} \left\{ \vec{r}_j \right\} \qquad \forall j = 1,..,J \tag{5}$$

EATWOS recurs to distance measures similar to known efficiency analysis techniques like DEA and TOPSIS. The distance measures op_{ij} for the outputs can be calculated on the basis of the matrix $\underline{R}$ and the maximum normalized output quantities r_j^*.

$$op_{ij} = 1 - (r_j^* - r_{ij}) \qquad \forall i = 1,...,I \qquad \forall j = 1,...,J \tag{6}$$

The distance measure op_{ij} is interpreted in the following way: the smaller the distance of r_{ij} to r_j^*, the closer op_{ij} is to one. This distance measure is taken as output score.

As a next step, the input quantities have to be normalized. The normalization of the input quantities takes place analogously to the output quantities.

$$\exists i \quad \exists k \quad x_{ik} \neq 0: \qquad s_{ik} = \frac{x_{ik}}{\sqrt{\sum_{i=1}^{I} x_{ik}^2}} \qquad \forall i = 1,..,I \quad \forall k = 1,...,K \tag{7a}$$

$$\forall i = 1,..,I \quad \forall k = 1,...,K \quad x_{ik} = 0: \qquad s_{ik} = 0 \tag{7b}$$

Thus, the normalized input matrix $\underline{S}$ is calculated analogously to the normalized output matrix:

$$\underline{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1k} & \cdots & s_{1K} \\ \cdots & \cdots & & \cdots & & \cdots \\ s_{i1} & s_{i2} & \cdots & s_{ik} & \cdots & s_{iK} \\ \cdots & \cdots & & \cdots & & \cdots \\ s_{I1} & s_{I2} & \cdots & s_{Ik} & \cdots & s_{IK} \end{bmatrix} \tag{8}$$

Subsequently, the minimum normalized input quantity s_k^* is determined for each input k on basis of the column vectors $\vec{s}_k$ of the normalized input matrix $\underline{S}$.

$$s_k^* = \min_{\mathrm{i}} \{ \vec{s}_k \} \qquad \forall k = 1,..,K \tag{9}$$

Then, the distance measure for inputs can be calculated by adding the respective value s_{ik} from the matrix $\underline{S}$ to 1 and subtracting the minimum normalized input quantity s_k^*.

$$ip_{ik} = 1 + s_{ik} - s_k^* \qquad \forall i = 1,...,I \qquad \forall k = 1,...,K \tag{10}$$

This distance measure is interpreted as follows: the smaller the distance of s_{ik} to s_k^*, the closer ip_{ik} is to one. The value 1 is added to avoid the distance measure ip_{ik} becoming zero. Analogously to the output score, the distance measure ip_{ik} is taken as input score.

The input distance measures (input score) and the output distance measures (output score) can be utilized to derive an efficiency score for each DMU. This efficiency score has the classical form in which the output scores are included in the numerator and the input scores are considered in the denominator. This kind of efficiency scores has been widely used in the literature. For example, the Data Envelopment Analysis (DEA) utilizes a similar efficiency score (e.g. Charnes, Cooper, and Thrall 1991, p. 198).

$$E_i = \frac{\sum_{j=1}^{J} v_j * op_{ij}}{\sum_{k=1}^{K} w_k * ip_{ik}} \qquad \forall i = 1,...,I \tag{11}$$

A low efficiency score E_i of a DMU i indicates a low efficiency relative to the other DMUs, while a high efficiency score E_i means a high efficiency. Thus, based on these efficiency scores, a rank order R of the efficiency of the DMUs can be derived by sorting the efficiency scores from high to low.

3.3 Application of EATWOS with consideration satisficing levels

The third step is to apply EATWOS with consideration of satisficing levels SL_j for at least one of the outputs j with $j \in \{1,...,J\}$. The outputs without satisficing levels are treated as described in the previous section. The idea of "satisficing" is incorporated into the efficiency analysis technique by embodying the following logical rule:

> If the output quantity y_{ij} of a DMU i exceeds a certain satisficing level SL_j, then the DMU receives the same output score as a DMU of which the output quantity is equal to the satisficing level.

In order to model this rule, five logical constraints are utilized. The idea for the linear representation of this logical constraints stems from Yan, Yu, and Cheng (2003, pp. 2143). For all outputs for which the decision maker determines satisficing levels, these following five constraints are applied:

[1a] $$\left(\frac{SL_j - y_{ij}}{SL_j}\right) + z_1 \leq 1$$

[1b] $$\left(\frac{SL_j - y_{ij}}{SL_j}\right) * z_2 \geq 0$$

[2] $$z_1, z_2 \in \{0;1\}$$

[3] $$z_1 + z_2 = 1$$

[4] $$a_{ij} = \frac{y_{ij}}{SL_j} * z_2 + 1 * z_1 = f(y_{ij})$$

The constraints [1a] and [1b] serve to restrict the possible values of the logical variables. Constraint [2] defines the logical variables z_1, z_2 as binary variables. Constraint [3] ensures in connection with constraint [2] that only one of the logical variables can take the value one, while the other one takes the value zero:

$$z_1 = 1 \Rightarrow z_2 = 0 \wedge z_1 = 0 \Rightarrow z_2 = 1$$

$$z_2 = 1 \Rightarrow z_1 = 0 \wedge z_2 = 0 \Rightarrow z_1 = 1$$

The possible values of the logical variables in constraint [4] are determined through the constraints [1a], [1b], [2], and [3]. Depending on the values of an output quantity y_{ij} and an accompanying satisficing level SL_j four value combinations are possible:

a.) The output quantity takes the value zero ($y_{ij} = 0$):

[1a] $$\left(\frac{SL_j - 0}{SL_j}\right) + z_1 \le 1$$

$$\Rightarrow 1 + z_1 \le 1 \quad \Rightarrow z_1 = 0$$

$$z_1 = 0 \quad \wedge \quad [3] \quad \Rightarrow \quad z_2 = 1$$

[1b] $$\left(\frac{SL_j - 0}{SL_j}\right) * 1 \ge 0$$

$$\Rightarrow 1 \ge 0 \qquad \checkmark$$

[4] $$\Rightarrow a_{ij} = \frac{0}{SL_j} * 1 + 1 * 0 = 0$$

b) The output quantity takes a value between zero and the satisficing level ($0 < y_{ij} < SL_j$):

[1a] $$]0;1[+ z_1 \le 1 \qquad \| \; z_1 \in \{0;1\} \quad \text{because of [2]}$$

$$\Rightarrow z_1 = 0$$

$$z_1 = 0 \quad \wedge \quad [3] \quad \Rightarrow \quad z_2 = 1$$

[1b] $$]0;1[* 1 \ge 0$$

$$\Rightarrow]0;1[\ge 0 \qquad \checkmark$$

[4] $$\Rightarrow a_{ij} = \frac{y_{ij}}{SL_j} * 1 + 1 * 0 = \frac{y_{ij}}{SL_j}$$

c) The output quantity is equal to the satisficing level ($y_{ij} = SL_j$):

[1a] $$\frac{SL_j - SL_j}{SL_j} + z_1 \le 1 \qquad \| \; z_1 \in \{0;1\} \quad \text{because of [2]}$$

$$\Rightarrow z_1 \in \{0;1\}$$

[1b] $$\frac{SL_j - SL_j}{SL_j} * z_2 \ge 0 \qquad \| \; z_2 \in \{0;1\} \quad \text{because of [2]}$$

$$\Rightarrow z_2 \in \{0;1\}$$

Because of [3], [1a] and [1b] are consistent with two alternative cases yielding to the same value of a_{ij} .

$$[4] \quad \left.\begin{array}{l} \text{c1)} \quad z_1 = 0 \quad \wedge \quad z_2 = 1 \\ \qquad a_{ij} = \frac{SL_j}{SL_j} * 1 + 1 * 0 = 1 \\ \vee \ \text{c2)} \quad z_1 = 1 \quad \wedge \quad z_2 = 0 \\ \qquad a_{ij} = \frac{SL_j}{SL_j} * 0 + 1 * 1 = 1 \end{array}\right\} \Rightarrow a_{ij} = 1$$

d) The output quantity is greater than the satisficing level ($y_{ij} > SL_j$):

$$[1a] \quad]-\infty; 0[+ z_1 \leq 1 \qquad \| \; z_1 \in \{0; 1\} \quad \text{because of [2]}$$

$$\Rightarrow z_1 \in \{0; 1\}$$

$$[1b] \quad]-\infty; 0[* z_2 \geq 0$$

$$\Rightarrow z_2 = 0$$

$$z_2 = 0 \quad \wedge \quad [3] \quad \Rightarrow \quad z_1 = 1$$

$$[4] \quad \Rightarrow a_{ij} = \frac{y_{ij}}{SL_j} * 0 + 1 * 1 = 1$$

The normalized output quantities a_{ij} are obtained by applying the constraints [1a], [1b], [2], [3], and [4], if a satisficing level SL_j is determined for the respective output j. These quantities are needed to compose the normalized output matrix $\underline{A}$. However, if no satisficing level is established for an output j, the respective column vector $\vec{a}_j$ in the matrix $\underline{A}$ is equal to the column vector $\vec{r}_j$ in the matrix $\underline{R}$.

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1j} & \ldots & a_{1J} \\ \ldots & \ldots & & \ldots & & \ldots \\ a_{i1} & a_{i2} & \ldots & a_{ij} & \ldots & a_{iJ} \\ \ldots & \ldots & & \ldots & & \ldots \\ a_{I1} & a_{I2} & \ldots & a_{Ij} & \ldots & a_{IJ} \end{bmatrix} \tag{12}$$

Subsequently, the maximum normalized output quantity a_j^* is determined for each output j by taking the maximum value of each column vector $\vec{a}_j$.

$$a_j^* = \max_{\text{i}} \{\vec{a}_j\} \qquad \forall j = 1,..,J \tag{13}$$

The maximum normalized output quantity a_j^* serves to calculate the distance measures for outputs. This distance measure is calculated for all DMUs i and for all outputs j.

$$op_{ij}^{SL} = 1 - (a_j^* - a_{ij}) \qquad \forall i = 1,...,I \qquad \forall j = 1,...,J \tag{14}$$

As in the second step, an efficiency score is calculated for each DMU. But in contrast to the efficiency score E_i calculated in the second step, the efficiency score E_i^{SL} incorporates the distance measures op_{ij}^{SL}, so that the satisficing levels for the outputs are considered.

$$E_i^{SL} = \frac{\sum_{j=1}^{J} v_j * op_{ij}^{SL}}{\sum_{k=1}^{K} w_k * ip_{ik}} \qquad \forall i = 1,...,I \tag{15}$$

Just as in the second step, a rank order R^{SL} of the efficiency of the DMUs can be obtained once again by sorting the efficiency scores E_i^{SL} from high to low.

3.4 Identifying efficiency improvement potentials

Efficiency improvement potentials can be identified by comparing the rank order R with the rank order R^{SL}. If the rank order R^{SL} based on the efficiency scores incorporating the satisficing levels differs from the rank order R, this is an indication that the efficiency of a DMU ranked lower with respect to rank order R than with rank order R^{SL} could be improved. This idea can be best illustrated by a simple example. Imagine two DMUs each characterized by one output quantity (y_1 for DMU 1, y_2 for DMU 2) and one input quantity (x_1 for DMU 1, x_2 for DMU 2). In this case, a simple ratio of the output quantity to the input quantity can be chosen as the efficiency score. It is assumed that DMU 1 is more efficient than DMU 2. Moreover, it is assumed that the output y_1 of DMU 1 is greater than the output y_2 of DMU 2, while y_2 is equal to the satisficing level SL.

$$\frac{y_1}{x_1} > \frac{y_2}{x_2} \qquad \text{with } y_1 > y_2 = SL \tag{16}$$

If the satisficing level is to be incorporated, the output quantities y_1 and y_2 can be replaced by the satisficing level, because the condition $y_1 > y_2 = SL$ holds. In this example, it is assumed that the rank order changes.

$$\frac{SL}{x_1} < \frac{SL}{x_2} \tag{17}$$

Therefore, DMU 2 has a higher rank in R^{SL} (rank 1) than in R (rank 2) due to the consideration of the satisficing level. Thus, it is possible that there are opportunities to improve the efficiency of DMU 2. Generally, four alternatives to improve the efficiency of a DMU can be differentiated.

1.) The first alternative is to increase the output quantity ($\Delta y_2 > 0$), while holding the input quantity constant ($\Delta x_2 = 0$).

2.) The second alternative is to increase the output quantity ($\Delta y_2 > 0$) and to increase the input quantity ($\Delta x_2 > 0$), while ensuring that the amount $|\Delta y_2|$ of increasing the output quantity is greater than the amount $|\Delta x_2|$ of increasing the input quantity.

3.) Another alternative is to decrease the input quantity ($\Delta x_2 < 0$) without decreasing the output quantity ($\Delta y_2 \geq 0$).

4.) The last alternative is to decrease the input quantity ($\Delta x_2 < 0$) and to decrease the output quantity ($\Delta y_2 < 0$), while ensuring that the amount $\left|\Delta y_2\right|$ of the reduction of the output quantity is smaller than the amount $\left|\Delta x_2\right|$ of the reduction of the input quantity.

In the given example, if the satisficing level is considered, increasing the output quantity ($\Delta y_2 > 0$) leads to no efficiency improvement, since y_2 is equal to the satisficing level. Thus, the efficiency improvement alternatives 1.) and 2.) cannot be applied to improve the efficiency of DMU 2. The key to achieving efficiency gains lies in the alternatives 3.) and 4.). The following equation elucidates the potential of efficiency improvement in the given example.

$$\frac{y_1}{x_1} = \frac{y_2 + \Delta y_2}{x_2 + \Delta x_2} \qquad \text{with } \Delta y_2 \leq 0 \quad \Rightarrow \quad \left|\Delta y_2\right| < \left|\Delta x_2\right| \tag{18}$$

According to equation (18), to increase the efficiency of DMU 2 the amount $\left|\Delta y_2\right|$ of the reduction of the output quantity has to be smaller than the amount $\left|\Delta x_2\right|$ of the reduction of the input quantity. If it is assumed that it is not desirable to reduce the output quantity ($\Delta y_2 = 0$), the maximum efficiency improvement is realized for:

$$\Delta x_2 = y_2 \frac{x_1}{y_1} - x_2$$

If this equation holds, the efficiency score of DMU 2 is the same as the efficiency score of DMU 1 without consideration of the satisficing level.

4. Numerical Example

4.1 Introduction

In practice, there are many cases in which it is appropriate to apply EATWOS. Consider, for example, an airline that has to make an investment decision between two or more airplanes from different manufacturers. The airline is capable of determining a maximum amount of air passengers on each flight and it is satisficed with this amount. This means that the airline will not try to increase the amount of passengers through an aggressive marketing campaign. Then, the maximum amount of passengers on each flight could be regarded as satisficing level. Another example might be an automotive manufacturer with two plants. If a particular car model is manufactured in both plants and the manufacturer cannot sell enough cars of this model, there are various ways to cope with this sales difficulties. If the manufacturer regards the closure of one plant as the only opportunity, the satisficing level for the output “number of cars” corresponds to the quantity of sales of this car model.

The numerical example presented in this section, however, is a case taken from a heat treatment company in southern Germany.

4.2 Input data preparation for the furnace efficiency analysis problem

The company has three different heat treatment furnaces for gas-nitriding, as shown in table 1. Gas-nitriding is a heat treating method mainly used to harden the surface of metal parts.

furnace	type of furnace	maximum connected rating [kilowatt]
1	shaft retort furnace	75
2	chamber retort furnace	64
3	chamber retort furnace	77

Table 1: Heat treatment furnaces

Three gases – ammonia gas, ammonia cracked gas and nitrogen have to be injected into the furnaces during the gas-nitriding process. Moreover, each furnace needs electricity, which is mainly consumed for the heating.

furnace	ammonia gas consumption [gallons per hour]	costs of ammonia gas consumption [€ per hour]	ammonia cracked gas consumption [gallons per hour]	costs of ammonia cracked gas consumption [€ per hour]	nitrogen consumption [gallons per hour]	costs of nitrogen consumption [€ per hour]	average electricity consumption [kilowatt hour]	costs of average electricity consumption [€ per hour]
		0.0042 € per gallon ammonia gas		[0.0025 € per gallon ammonia cracked gas]		[0.0006 € per gallon nitrogen]		[0.09 € per kilowatt hour]
1	370	1.55	53	0.13	106	0.06	22	1.98
2	291	1.22	79	0.20	159	0.10	20	1.80
3	343	1.44	105	0.26	185	0.11	25	2.25

Table 2: Consumption and costs of gases and electricity per hour

The costs in table 2 are aggregated to one input, as can be seen in the second column of table 3. The second input is the amount of work for loading and unloading of a furnace. The priorities in the third column of table 3 can be derived by employing a simple scoring technique or more elaborated techniques like the AHP. The priority of 0.750 for furnace 1 means that the amount of work required for the loading and unloading of furnace 1 is greater than that of the other two furnaces. This is because furnace 1 is a shaft retort furnace, which must be loaded from the top, while the chamber retort furnaces can be loaded from the front. Furthermore, the shaft retort furnace requires the worker to secure and release various bolt mechanisms in order to close and then reopen the furnace. A chamber furnace, however, can be closed by simply locking the door. The maximum charge weight and the packing space are selected as outputs for the efficiency analysis. Since the charges are never heavier than 350 kilograms and usually the company has only to cope with at most one charge per day, the satisficing level for the output maximum charge weight is set to 350 ($SL_1 = 350$).

	Inputs		Outputs	
furnace	total costs per hour (= total sum of table 2) [€ per hour]	amount of work for loading and unloading of the furnace [priorities]	maximum charge weight [kilograms]	packing space [cubic inch]
1	3.72 (=1.55+0.13+0.06+1.98)	0.750	300	23348
2	3.32 (=1.22+0.20+0.10+1.80)	0.125	350	21960
3	4.06 (=1.44+0.26+0.11+2.25)	0.125	650	26169

Table 3: Inputs and outputs efficiency analysis

Furthermore, the decision maker has to establish the relative importance weights v_j of the outputs as well as the relative importance weights w_k of the inputs. As outlined above, the decision maker can do this by employing a scoring technique. In this case, the decision maker has judged the total cost per hour as more important than the amount of work for loading and unloading the furnaces (see table 4). Moreover, the decision maker assumes the outputs to be equally important.

relative importance weights w_k of the inputs		relative importance weights v_j of the outputs	
Total costs per hour	amount of work for loading and unloading of the furnace	maximum charge weight	packing space
0.6	0.4	0.5	0.5

Table 4: Relative importance weights

4.3 Application of EATWOS without consideration of satisficing levels to the furnace selection problem

Firstly, the output quantities have to be normalized. This is shown exemplary for r_{11}:

$$r_{11} = \frac{300}{\sqrt{300^2 + 350^2 + 650^2}} \approx 0.38$$

Then, the decision maker obtains the normalized output matrix $\underline{R}$:

$$\underline{R} = \begin{bmatrix} 0.38 & 0.56 \\ 0.44 & 0.53 \\ 0.82 & 0.63 \end{bmatrix}$$

On basis of the column vectors of $\vec{r}_j$ of the normalized output matrix $\underline{R}$ the maximum normalized output quantities r_1^* and r_2^* can be determined:

$$r_1^* = 0.82 \quad \wedge \quad r_2^* = 0.63$$

After that, the decision maker is able to calculate the six output distance measures:

$$op_{11} = 1-(0.82-0.38) = 0.56 \qquad op_{12} = 1-(0.63-0.56) = 0.93$$

$$op_{21} = 1-(0.82-0.44) = 0.62 \qquad op_{22} = 1-(0.63-0.53) = 0.90$$

$$op_{31} = 1-(0.82-0.82) = 1.00 \qquad op_{32} = 1-(0.63-0.63) = 1.00$$

Then, the input quantities have to be normalized. The result of the normalization process is the following normalized input matrix $\underline{S}$:

$$\underline{S} = \begin{bmatrix} 0.58 & 0.97 \\ 0.52 & 0.16 \\ 0.63 & 0.16 \end{bmatrix}$$

Next, the minimum normalized input quantities s_1^* und s_2^* are determined:

$$s_1^* = 0.52 \quad \wedge \quad s_2^* = 0.16$$

Then, the six input distance measures are calculated:

$$ip_{11} = 1 + 0.58 - 0.52 = 1.06 \qquad ip_{12} = 1 + 0.97 - 0.16 = 1.81$$

$$ip_{21} = 1 + 0.52 - 0.52 = 1.00 \qquad ip_{22} = 1 + 0.16 - 0.16 = 1.00$$

$$ip_{31} = 1 + 0.63 - 0.52 = 1.11 \qquad ip_{32} = 1 + 0.16 - 0.16 = 1.00$$

The final stage in his step involves calculating the efficiency scores E_i .

$$E_1 = \frac{0.5 * 0.56 + 0.5 * 0.93}{0.6 * 1.06 + 0.4 * 1.81} \approx 0.55, \; E_2 = 0.76, \; E_3 = 0.94$$

Thus, the rank order R of efficiency is as follows: $E_3 \succ E_2 \succ E_1$

4.4 Application of EATWOS with consideration of satisficing levels to the furnace selection problem

The logical constraints, presented in section 3.3, have to be applied to the output maximum charge weight, since a satisficing level ($SL_1 = 350$) is determined for this output.

b) The output quantity of furnace 1 takes a value between zero and the satisficing level ($0 < 300 = y_{11} < 350 = SL_1$):

[1a] $$\left(\frac{350 - 300}{350}\right) + z_1 \le 1 \;\Rightarrow\; z_1 = 0 \;\Rightarrow\; z_2 = 1$$

[1b] $$\left(\frac{350 - 300}{350}\right) * 1 \ge 0 \qquad \checkmark$$

[4] $$a_{11} = \frac{300}{350} * 1 + 1 * 0 \approx 0,86$$

c) The output quantity of furnace 2 is equal to the satisficing level ($y_{21} = SL_1 = 350$):

[1a] $$\left(\frac{350 - 350}{350}\right) + z_1 \le 1$$

$$\Rightarrow z_1 \in \{0; 1\}$$

[1b] $$\left(\frac{350-350}{350}\right)*z_2 \geq 0$$

$$\Rightarrow z_2 \in \{0;1\}$$

[4]
$$\left.\begin{array}{l} \text{c1)}\quad z_1 = 0 \quad \wedge \quad z_2 = 1 \\ \qquad a_{21} = \frac{350}{350}*1+1*0 = 1 \\ \vee\ \text{c2)}\quad z_1 = 1 \quad \wedge \quad z_2 = 0 \\ \qquad a_{21} = \frac{350}{350}*0+1*1 = 1 \end{array}\right\} \Rightarrow a_{21} = 1$$

d) The output quantity of furnace 3 is greater than the satisficing level ($y_{31} = 650 > 350 = SL_1$):

[1a] $$\left(\frac{350-650}{350}\right)+z_1 \leq 1$$

$$\Rightarrow z_1 \in \{0;1\}$$

[1b] $$\left(\frac{350-650}{350}\right)*z_2 \geq 0$$

$$\Rightarrow z_2 = 0$$

$$z_2 = 0 \quad \wedge \quad [3] \quad \Rightarrow \quad z_1 = 1$$

[4] $$a_{31} = \frac{650}{350}*0+1*1 = 1$$

The normalized output quantities a_{11}, a_{21}, and a_{31} are entered in the normalized output matrix $\underline{A}$. The column vector $\vec{a}_2$ corresponds to the column vector $\vec{r}_2$, since for output 2 no satisficing level was considered.

$$\underline{A} = \begin{bmatrix} 0.86 & 0.56 \\ 1.00 & 0.53 \\ 1.00 & 0.63 \end{bmatrix}$$

Subsequently, the maximum normalized output quantities a_1^* and a_2^* are determined on the basis of matrix $\underline{A}$.

$$a_1^* = 1.00 \quad \wedge \quad a_2^* = r_2^* = 0.63$$

Then, the output distance measures for the output maximum charge weight are calculated to consider the satisficing level for this output:

$$op_{11}^{SL} = 1-(1.00-0.86) = 0.86$$

$$op_{21}^{SL} = 1 - (1.00 - 1.00) = 1.00$$

$$op_{31}^{SL} = 1 - (1.00 - 1.00) = 1.00$$

The op_{i2}-values can be taken from section 4.3, since no satisficing level has been specified for output 2. Finally, the efficiency scores E_i^{SL} incorporating the satisficing level for the output maximum charge weight have to be calculated.

$$E_1^{SL} = \frac{0.5*0.86 + 0.5*0.93}{0.6*1.06 + 0.4*1.81} \approx 0.66,\ E_2^{SL} = 0.95,\ E_3^{SL} = 0.94$$

Hence, the rank order R^{SL} of efficiency incorporating the satisficing level SL_1 is as follows:

$$E_2^{SL} \succ E_3^{SL} \succ E_1^{SL}$$

Furnace 2 has a higher rank in R^{SL} (rank 1) than in R (rank 2) due to the consideration of the satisficing level.

4.5 Identifying efficiency improvement potentials of the second furnace

Since the rank order R differs from the rank order R^{SL} and furnace 2 has with respect to R a lower relative efficiency than with respect to R^{SL}, there are possibly ways to improve the efficiency of the second furnace. The decision maker has to look for variations of the input quantities and output quantities of furnace 2, which can yield efficiency gains. The decision maker cannot realize efficiency gains by increasing the output quantity y_{21} since the maximum charge weight is already equal to the satisficing level SL_1. Moreover, it can be assumed that it is technically impossible or at least too expensive to change the packing space of a furnace. That means that the output quantity y_{22} cannot be changed. Hence, the decision maker has to turn to the input quantities. The input “amount of work for loading and unloading of the furnace” offers no potential to improve the relative efficiency of furnace 2 in comparison to furnace 3 since the furnaces are of the same type. Thus, the only way to improve the relative efficiency of furnace 2 is to reduce the input quantity x_{21}. For example, it is possible that furnace 2 has leakages and thus consumes too much gas. In this case, a cost reduction can be achieved by improving the atmospheric integrity of the furnace and hence lowering the gas consumption. Another possibility is that furnace 2 consumes too much electricity, since its heating unit has to be modernized. In the best case scenario, the efficiency of furnace 2 is improved so much that it reaches the relative efficiency of furnace 3 ($E_2^{'} = E_3^{'}$) without considering the satisficing level for output 1. For this, the decision maker has to lower the input quantity x_{21} to 1.11 €. He then obtains the following efficiency scores:

$$E_1^{'} = 0.46,\ E_2^{'} = E_3^{'} = 0.76$$

5. Concluding Remarks

In the present paper, EATWOS is proposed as a new approach to efficiency analysis. It combines classical concepts of efficiency analysis, such as distance measures and efficiency scores with Simon's idea of satisficing into a new type of efficiency analysis technique. As has been shown, if a satisficing level is defined for at least one output, the proposed efficiency analysis technique can lead to different efficiency scores and thus to a different rank order of the DMUs in comparison to known efficiency analysis techniques. The application of EATWOS can assist decision makers in rethinking serious investment decisions and such grave decisions as plant closures. If a decision maker is conscious of satisficing levels for output quantities, he may apply EATWOS to consider these satisficing levels and thus avoid misleading decisions.

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