

A formal model for production order selection considering synergies

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A formal model for production order selection

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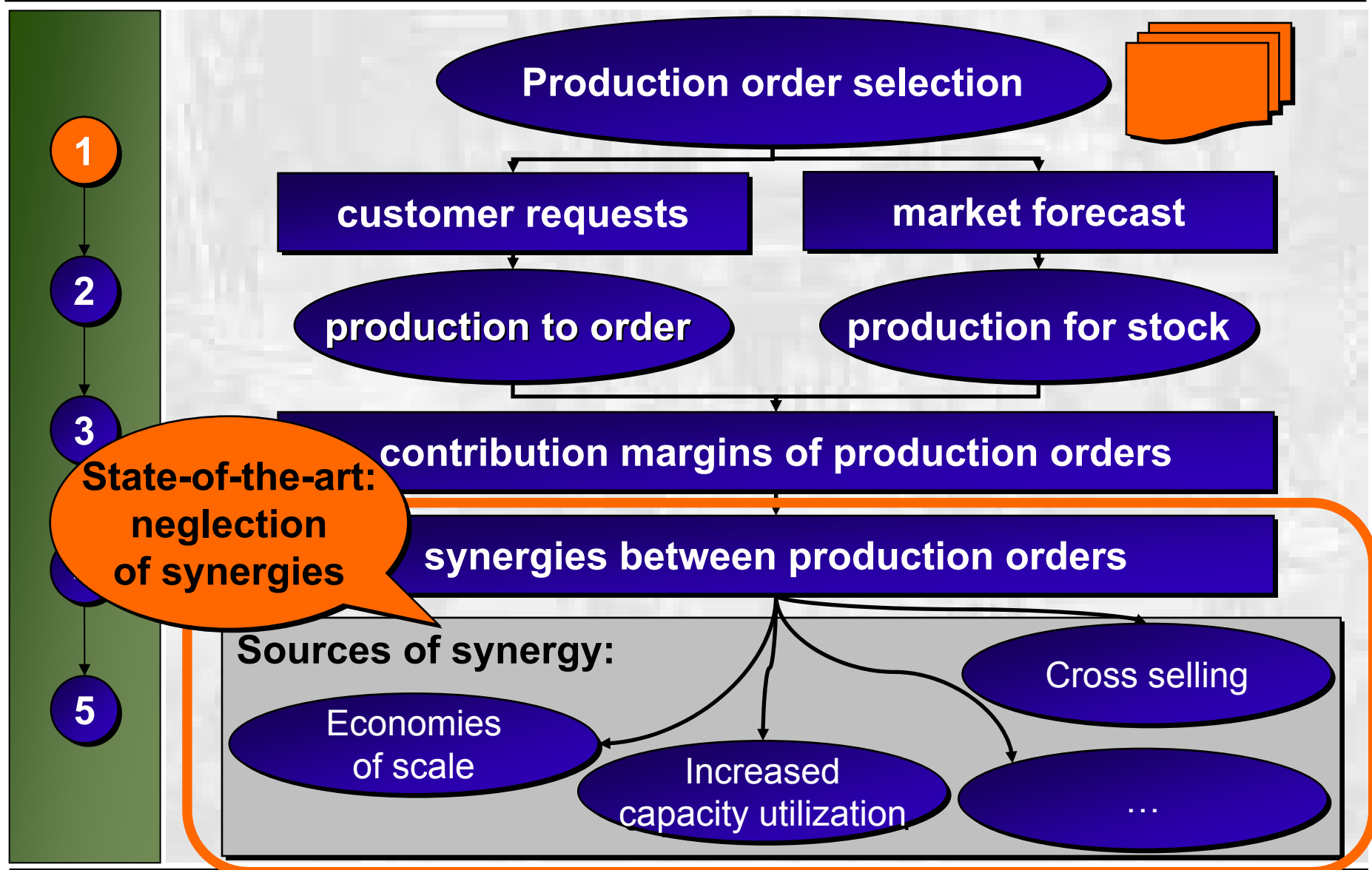
Numerical Example

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Difficulties

A formal model for production order selection

Problem identification



A formal model for production order selection

Input data preparation

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Available capacity of each type of capacity



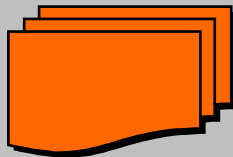
Example: Capacity type “machine saw”

4 machine saws à 60 h operating time per week

Available capacity of capacity type “machine saw” per week:

$$4 * 60 \text{ h} = 240 \text{ h}$$

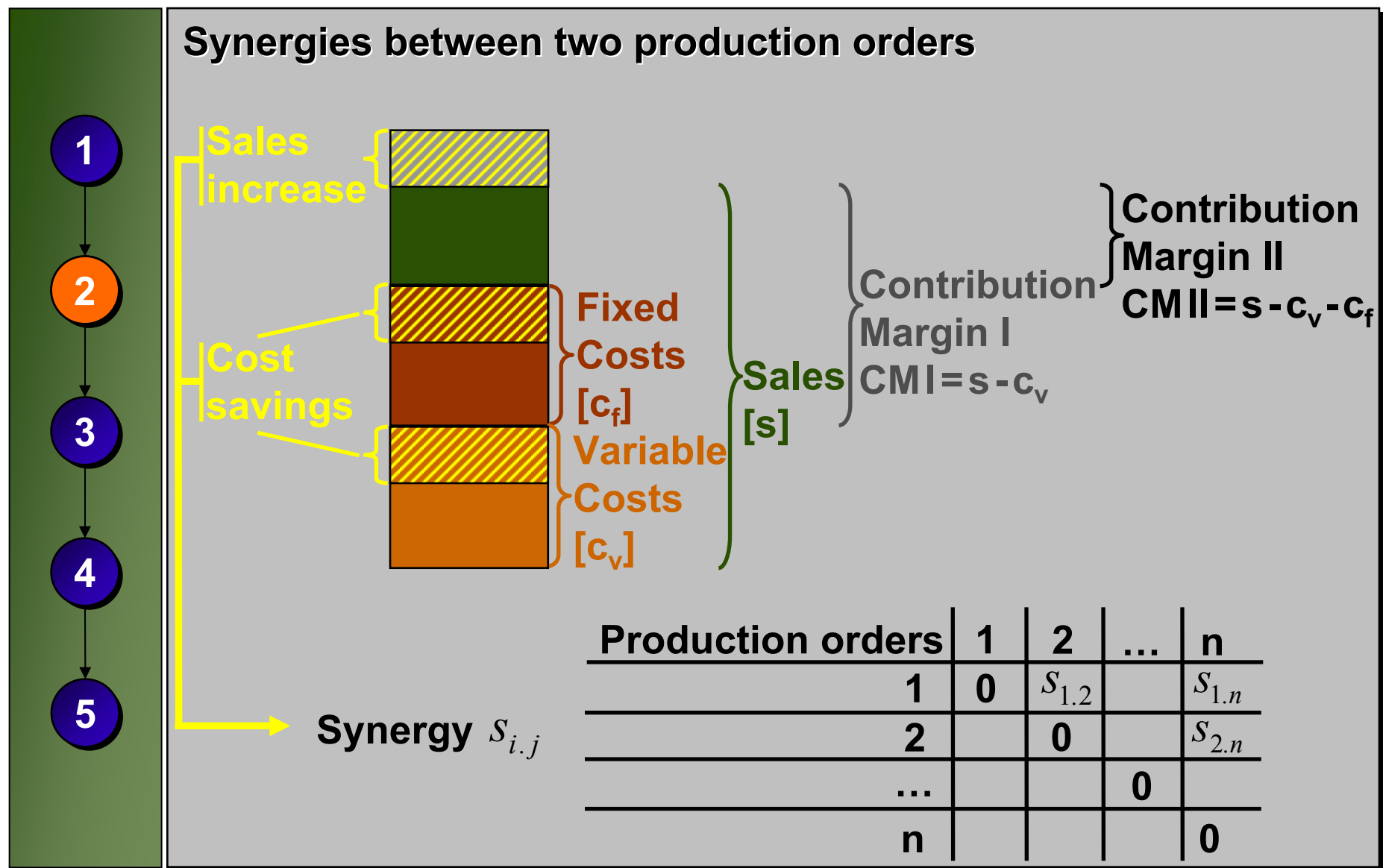
Production orders



- Required capacity of each type of capacity
- Contribution Margins

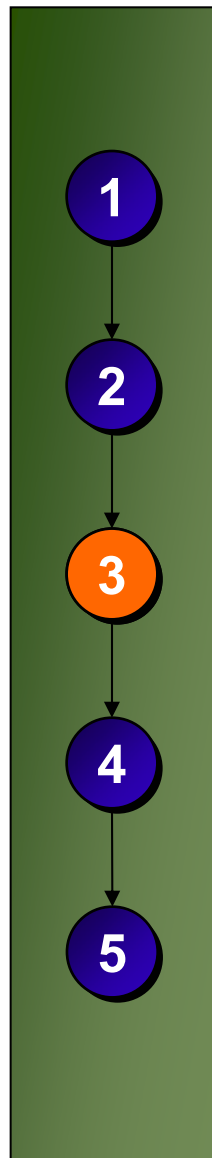
A formal model for production order selection

Input data preparation

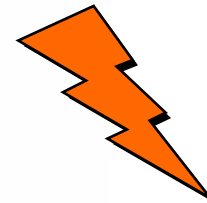


A formal model for production order selection

Problem Solution – Simple knapsack model



- n number of potential production orders
- M_i contribution margin of production order i
- u_i binary variable,
 $u_i = 1$, if production order i is selected.
 $u_i = 0$, if production order i is not selected.
- c_i required capacity for production order i
- C available capacity



single type of capacity

neglect of synergies



Extended knapsack model

Objective function:

$$\text{Max } \sum_{i=1}^n u_i * M_i$$

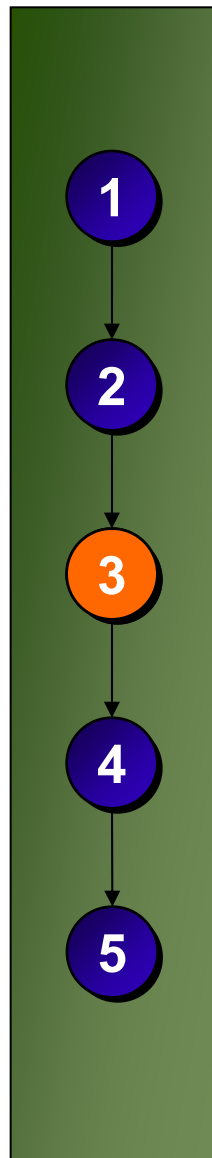
subject to the constraints:

$$\sum_{i=1}^n c_i * u_i \leq C$$

$$u_i \in \{0, 1\} \quad \forall i = 1, \dots, n$$

A formal model for production order selection

Problem Solution – Extended knapsack model



$c_{i,k}$ required capacity of type k for production order i

C_k available capacity of type k

Different types of capacity

K number of types of capacities

u_j binary variable, **Second binary variable**

$u_j = 1$, if production order j is selected

$u_j = 0$, if production order j is not selected

$s_{i,j}$ synergy, if production order i and production order j are selected

Synergies

Objective function:

$$\text{Max} \sum_{i=1}^n u_i * M_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ j>i}}^n s_{i,j} * u_i * u_j$$

subject to the constraints:

$$\sum_{i=1}^n c_{i,k} * u_i \leq C_k \quad \forall k = 1, \dots, K$$

$$u_i \in \{0, 1\} \quad \forall i = 1, \dots, n$$

$$u_j \in \{0, 1\} \quad \forall j = 1, \dots, n$$

A formal model for production order selection

Numerical Example – without consideration of synergies

- 1
- 2
- 3
- 4
- 5

MODEL:

```

SETS:
  PRODUCTION_ORDERS / pror_1, pror_2, pror_3, pror_4, pror_5/:
    PROR_SELECT, CONTRIBUTION_MARGIN;
  COMPETENCIES/ comp_1, comp_2, comp_3, comp_4, comp_5/: CAPA_AVAILABLE;
  CAPACITY_PROR(PRODUCTION_ORDERS, COMPETENCIES): CAPA_REQUIRED;
  PROR_SYNERGIES (PRODUCTION_ORDERS, PRODUCTION_ORDERS): SYNERGY;
ENDSETS

DATA:
  CONTRIBUTION_MARGIN =
    215
    280
    220
    190
    220;

  CAPA_REQUIRED =
    0 0 0 0 0
    0 0 0 0 0
    0 0 0 0 0
    0 0 0 0 0
    0 0 0 0 0;

  CAPA_AVAILABLE =
    0 0 0 0 0
    0 0 0 0 0
    0 0 0 0 0
    0 0 0 0 0
    0 0 0 0 0;

  SYNERGY = 0 0 0 0 0
            0 0 0 0 0
            0 0 0 0 0
            0 0 0 0 0
            0 0 0 0 0;
ENDDATA
        
```

Solution Report - pos_igls_ws

Linearization components added:
 Constraints: 100
 Variables: 25
 Integers: 25

Global optimal solution found at iteration:
 Objective value: **630.0000**

Variable	Value	Reduced
PROR_SELECT(PROR_1)	0.000000	-215
PROR_SELECT(PROR_2)	0.000000	-280
PROR_SELECT(PROR_3)	1.000000	-220
PROR_SELECT(PROR_4)	1.000000	-190
PROR_SELECT(PROR_5)	1.000000	-220
CONTRIBUTION_MARGIN(PROR_1)	215.0000	0.0
CONTRIBUTION_MARGIN(PROR_2)	280.0000	0.0
CONTRIBUTION_MARGIN(PROR_3)	220.0000	0.0
CONTRIBUTION_MARGIN(PROR_4)	190.0000	0.0
CONTRIBUTION_MARGIN(PROR_5)	220.0000	0.0

production order
3, 4, 5

Variable	Value	Reduced
PROR_SELECT(PROR_1)	0.000000	-215
PROR_SELECT(PROR_2)	0.000000	-280
PROR_SELECT(PROR_3)	1.000000	-220
PROR_SELECT(PROR_4)	1.000000	-190
PROR_SELECT(PROR_5)	1.000000	-220
CONTRIBUTION_MARGIN(PROR_1)	215.0000	0.0
CONTRIBUTION_MARGIN(PROR_2)	280.0000	0.0
CONTRIBUTION_MARGIN(PROR_3)	220.0000	0.0
CONTRIBUTION_MARGIN(PROR_4)	190.0000	0.0
CONTRIBUTION_MARGIN(PROR_5)	220.0000	0.0

Screenshot: Lingo 8.0, Lindo Systems Inc.

A formal model for production order selection

Numerical Example – with consideration of synergies

- 1
- 2
- 3
- 4
- 5

MODEL:

SETS:

```

PRODUCTION_ORDERS / pror_1, pror_2, pror_3, pror_4, pror_5/:
  PROR_SELECT, CONTRIBUTION_MARGIN;
COMPETENCIES/ comp_1, comp_2, comp_3, comp_4, comp_5/: CAPA_AVAILABLE;
CAPACITY_PROR( PRODUCTION_ORDERS, COMPETENCIES): CAPA_REQUIRED;
PROR_SYNERGIES ( PRODUCTION_ORDERS, PRODUCTION_ORDERS): SYNERGY;
    
```

ENDSETS

DATA:

```

CONTRIBUTION_MARGIN =
    215
    280
    220
    190
    220;

CAPA_REQUIRED =
    25
    30
    10
    0;

CAPA_AVAILABLE =
    30
    30
    30
    30
    30;

SYNERGY = 0
    0 0
    0 0 0
    0 0 0 0
    0 0 0 0 0;
    
```

ENDDATA

LINGO - [Solution Report - pos_igls]

Linearization components added:

```

Constraints:    100
Variables:      25
Integers:       25
    
```

Global optimal solution found at iteration: 100
Objective value: **705.0000**

Variable	Value	Reduced Cost
PROR_SELECT(PROR_1)	1.000000	-215
PROR_SELECT(PROR_2)	0.000000	-280
PROR_SELECT(PROR_3)	1.000000	-220
PROR_SELECT(PROR_4)	1.000000	-190
PROR_SELECT(PROR_5)	0.000000	-220
CONTRIBUTION_MARGIN(PROR_1)	215.0000	0.0000
CONTRIBUTION_MARGIN(PROR_2)	280.0000	0.0000
CONTRIBUTION_MARGIN(PROR_3)	220.0000	0.0000
CONTRIBUTION_MARGIN(PROR_4)	190.0000	0.0000
CONTRIBUTION_MARGIN(PROR_5)	220.0000	0.0000

production order
1, 3, 4

Variable	Value	Reduced Cost
PROR_SELECT(PROR_1)	1.000000	-215
PROR_SELECT(PROR_2)	0.000000	-280
PROR_SELECT(PROR_3)	1.000000	-220
PROR_SELECT(PROR_4)	1.000000	-190
PROR_SELECT(PROR_5)	0.000000	-220
CONTRIBUTION_MARGIN(PROR_1)	215.0000	0.0000
CONTRIBUTION_MARGIN(PROR_2)	280.0000	0.0000
CONTRIBUTION_MARGIN(PROR_3)	220.0000	0.0000
CONTRIBUTION_MARGIN(PROR_4)	190.0000	0.0000
CONTRIBUTION_MARGIN(PROR_5)	220.0000	0.0000

Screenshot: Lingo 8.0, Lindo Systems Inc.

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Numerical Example – key customer production order

enforced selection of production order 2

production order 2, 5

Objective value: **550.0000**

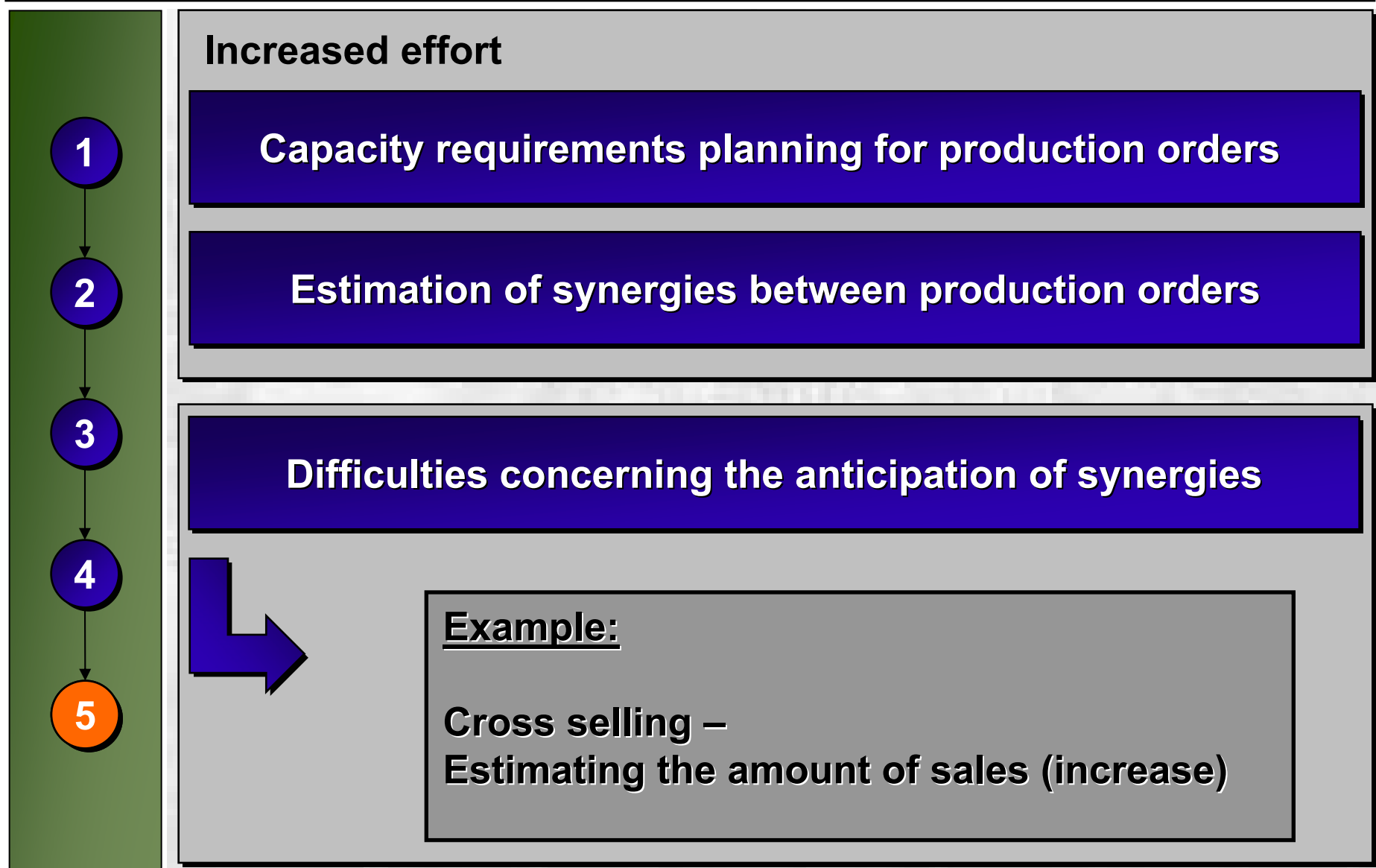
Variable	Value	Reduced
PROR_SELECT(PROR_1)	0.000000	-245
PROR_SELECT(PROR_2)	1.000000	0.0
PROR_SELECT(PROR_3)	0.000000	-265
PROR_SELECT(PROR_4)	0.000000	-227
PROR_SELECT(PROR_5)	1.000000	-270
CONTRIBUTION_MARGIN(PROR_1)	215.0000	0.0
CONTRIBUTION_MARGIN(PROR_2)	280.0000	0.0
CONTRIBUTION_MARGIN(PROR_3)	250.0000	0.0
CONTRIBUTION_MARGIN(PROR_4)	190.0000	0.0

PROR_SELECT (2) = 1;

Screen Shot: Lingo 8.0, Lindo Systems Inc.

A formal model for production order selection

Difficulties



A formal model for production order selection

1

2

3

4

5

Thank you for your time and attention!



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