Modeling of Fairness

– Distribution of Efficiency Gains in Supply Webs
from a Game-theoretic Point of View –

March 6, Innsbruck, Austria

Stephan Zelewski; Malte L. Peters

{ stephan.zelewski | malte.peters } @pim.uni-due.de
Institute for Production and Industrial Information Management
University of Duisburg-Essen, Campus Essen

Outline

1. Problem identification
2. Requirements to restrict the solution space
3. Calculation rule for τ-value
4. Concluding remarks

Problem identification

Supply chain management:

- Realizing efficiency gains by coordinating the activities of all supply chain / supply web participants
- …

Problems

- distribution of the efficiency gains among partially autonomous actors in a supply web
- each actor is interested in maximizing his own gain
- built-in conflict between cooperation and defection

Generic distribution problem:

- Find a distribution of efficiency gains ensuring that
  - the actors in a supply web are willing to cooperate with each other and that
  - the actors accept as “fair”

Problem identification

Standard approach in game theory:

- Specifying a set of “reasonable” axioms / applicability conditions of a solution concept
- Is the solution concept the only one that is capable of fulfilling a set of axioms?

Problems

- justification of the axioms
- difficult to detect the connection between axioms and the respective real problems

alternative approach:

- Starting from the real problem to distribute efficiency gains among the actors in a supply web
- The solution concept should be derivable from plausible requirements oriented towards the real problem

⇒ Definition of five requirements

Requirements to restrict the solution space

First requirement:
Individual rationality

- Perfect rationality: each actor maximizes her or his individual utility
  - no envy effects
  - information processing capacity of the actors is not restricted
  - …
- It would **not** be **rational** for an actor A_n to **participate in the supply web** within the coalition K_0, if this coalition yields a smaller utility in comparison to realizing the amount $c(\{A_n\})$ **outside the supply web.**
- The condition of individual rationality can be formulated with the function c as follows:

$$\forall L_v \in \mathbb{R}^N_{\geq 0}: \quad L_v = \begin{pmatrix} v_1 \\ \ldots \\ v_N \end{pmatrix} \geq \begin{pmatrix} c(\{A_1\}) \\ \ldots \\ c(\{A_N\}) \end{pmatrix}$$

→ solution space is restricted to the first quadrant

Requirements to restrict the solution space

Second requirement

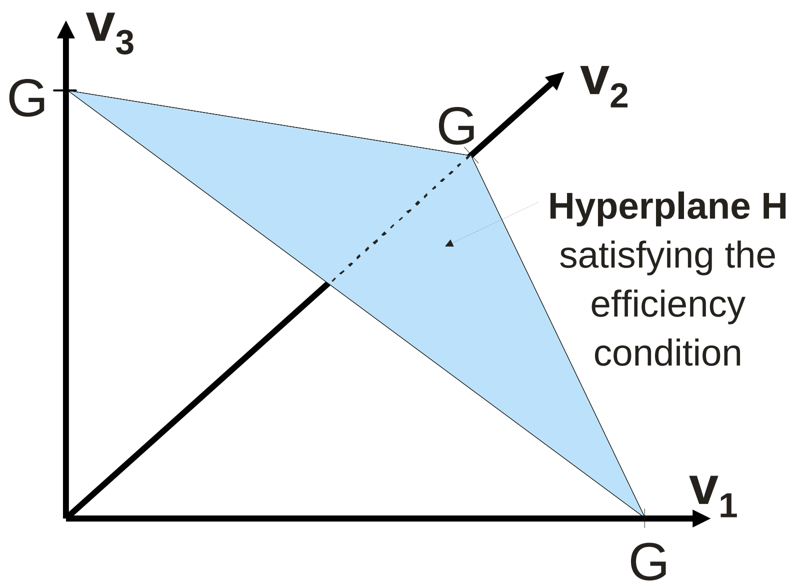
Efficiency condition

- The efficiency gain G is distributed completely among the coalition $K_0 = \{A_1,\ldots,A_N\}$ of all actors
  - It would be **irrational** to distribute less than the efficiency gain G
    → loss of Pareto Optimality
  - It would be **irrational** to distribute more than the efficiency gain G

$$\forall L_v \in \mathbb{R}^N_{\geq 0}: \quad L_v = \begin{pmatrix} v_1 \\ \ldots \\ v_N \end{pmatrix} \rightarrow \sum_{n=1}^{N} v_n = G$$

→ Hyperplane H as the set of all solutions that are satisfying the condition:

$$\sum_{n=1}^{N} v_n = G$$



Requirements to restrict the solution space

Third requirement
Rationality condition for maximum allocable shares of the gain

- The N-1 actors of the so called marginal coalition MK_n with $MK_n = K_0 \backslash \{A_n\}$ grant actor A_n at most the share $v_{n.max}$ of the efficiency gain G, if actor A_n would leave the great coalition $K_0 = \{A_1,...,A_N\}$, so that the efficiency gain G would decrease.

→ Rationality condition:

$$\forall n = 1,...,N \ \forall v_n \in \mathbb{R} : v_n \leq v_{n.max} \wedge v_{n.max} = c(K_0) - c(MK_n) = G - c(MK_n)$$

In the solution space the point, in which the maximum allocable share $v_{n.max}$ of the gain is assigned to each actor A_n, is called *upper bound* UB or *ideal point*.

Requirements to restrict the solution space

Fourth requirement
Rationality condition for minimal allocable shares of the gain

- Each actor A_n receives at least the share $v_{n.min}$ of the efficiency gain G, so that she or he could believably threaten to found an outsider coalition $AK_{n.q}$

- **Side payments:** Actor A_n makes an offer (= side payment) to the other actors in an outsider coalition $AK_{n.q}$.

  The side payment ensures that the utility of each other actor is the same as his or her maximum utility in the great coalition K_0.

  → The actors in an outsider coalition have no incentives to remain in the great coalition K_0.

Complete rationality condition for minimal allocable shares of the gain:

$$\forall n = 1,\dots,N \; \forall v_n \in \mathbb{R}_{\geq 0}: \quad v_n \geq v_{n.min} \;\wedge\; v_{n.min} = \max\{c_{n.1}, c_{n.2}, 0\}$$

with

$$c_{n.1} = \max\left\{ c\left(\{A_n\} \middle| AK_{n.q}\right) = c\left(AK_{n.q}\right) - \sum_{m \in \left(\mathbb{N}_{n.q} \setminus \{n\}\right)} v_{m.max} \;\middle|\; \begin{array}{c} \dots \\ \emptyset \subset AK_{n.q} \subset A \;\wedge\; \{A_n\} \subset AK_{n.q} \end{array} \right\}$$

$$c_{n.2} = c\left(\{A_n\} \middle| AK_{n.q}\right) = c\left(\{A_n\}\right) \text{ for } AK_{n.q} = \{A_n\}$$

→ lower bound LB / **threat point**

Fifth requirement

Operational fairness criterion

- The greater bargaining power of an actor A_n is, the greater is her or his share v_n of the efficiency gain G.
  - The bargaining power of an actor A_n is affected by two opposed effects:
    - On the one hand the bargaining power of an actor A_n is measured by the contribution $\mathbf{v_{n.max}}$ the actor would make, if she or he would take part in the marginal coalition MK_n and thus would make this marginal coalition a great coalition K_0.

      → positive network effect
    - On the other hand the bargaining power of an actor A_n is measured by her or his threat potential $\mathbf{v_{n.min}}$ build up of the believable threat to found an outsider coalition $AK_{n.q}$.

      → negative network effect

Calculation rule for τ-value

normalized bargaining powers

$$\forall n = 1,\dots,N:$$

$$v_{n.\tau} = \begin{cases} \alpha \cdot \dfrac{v_{n.max}}{\sum_{n=1}^{N} v_{n.max}} \cdot G + \beta \cdot \dfrac{v_{n.min}}{\sum_{n=1}^{N} v_{n.min}} \cdot G; & \text{if } \sum_{n=1}^{N} v_{n.max} \neq \sum_{n=1}^{N} v_{n.min} \\ v_{n.max} = v_{n.min} & ; \text{ if } \sum_{n=1}^{N} v_{n.max} = \sum_{n=1}^{N} v_{n.min} \end{cases}$$

with

Weighting of the bargaining powers

$$\alpha = \frac{G - \sum_{n=1}^{N} v_{n.min}}{\sum_{n=1}^{N} v_{n.max} - \sum_{n=1}^{N} v_{n.min}} \cdot \frac{\sum_{n=1}^{N} v_{n.max}}{G}$$

$$\beta = \frac{\sum_{n=1}^{N} v_{n.max} - G}{\sum_{n=1}^{N} v_{n.max} - \sum_{n=1}^{N} v_{n.min}} \cdot \frac{\sum_{n=1}^{N} v_{n.min}}{G}$$

Calculation rule for τ-value

Equivalent representation of the τ-value as

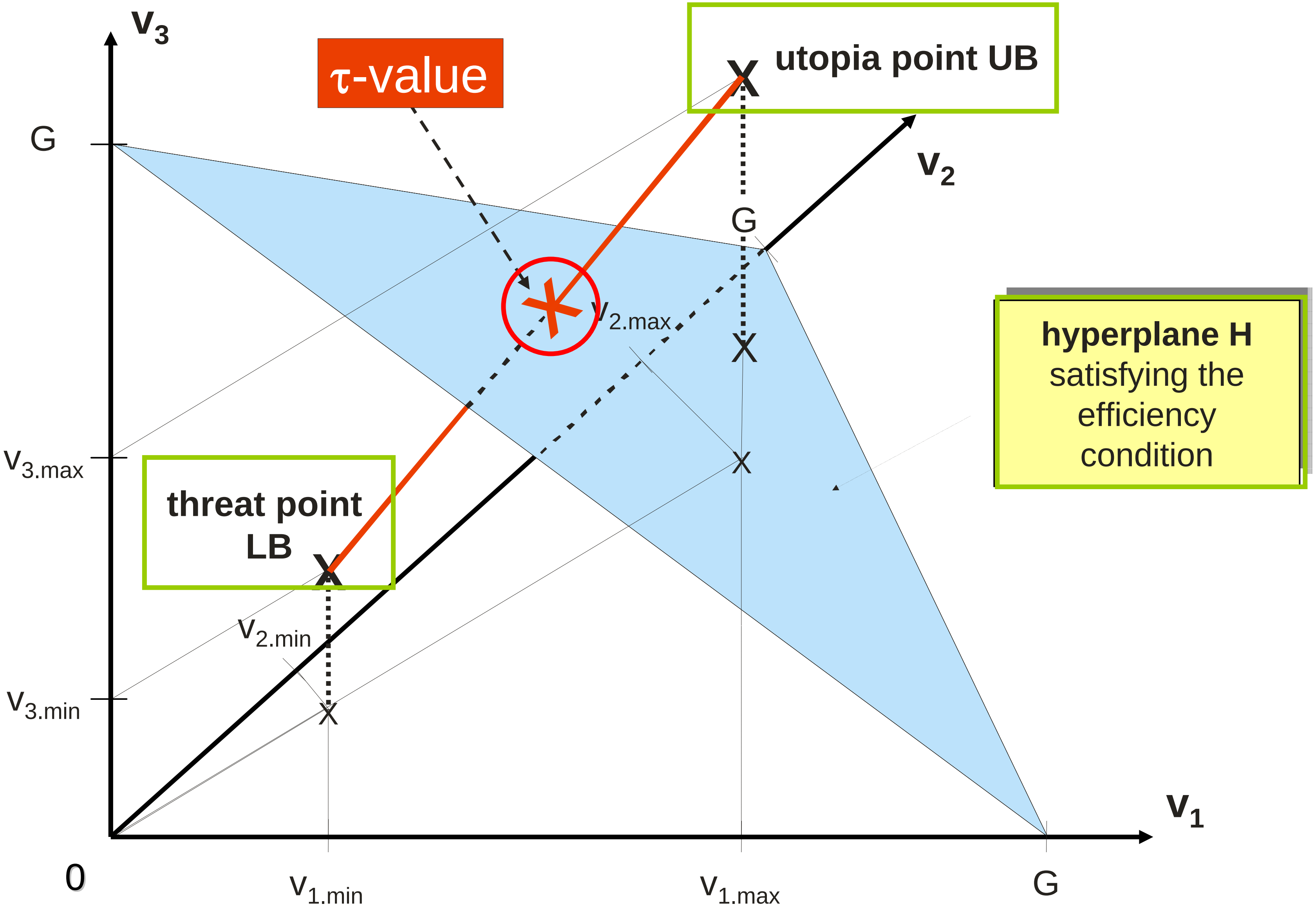
- linear combination of **utopia point** UB and **threat point** LB
- lying on the hyperplane H within the solution space and thus satisfying the efficiency condition

$$\forall n = 1,\dots,N: \quad v_{n.\tau} = \gamma \bullet v_{n.max} + (1-\gamma) \bullet v_{n.min}$$

with

$$\gamma = \begin{cases} \dfrac{G - \sum_{n=1}^{N} v_{n.min}}{\sum_{n=1}^{N} v_{n.max} - \sum_{n=1}^{N} v_{n.min}} & ;\ \text{if } \sum_{n=1}^{N} v_{n.max} \neq \sum_{n=1}^{N} v_{n.min} \\ 0 \vee 1 & ;\ \text{if } \sum_{n=1}^{N} v_{n.max} = \sum_{n=1}^{N} v_{n.min} \end{cases}$$

Concluding remarks



Concluding remarks

+ τ-value solution concept fulfills the five requirements

+ good reasons to accept a distribution of the efficiency gain G among the N actors as **fair**
  - share of the efficiency gain depends on the actor's bargaining power
  - compromise solution
  - distribution result is explicit

+ τ-value solution concept can be communicated easily in comparison to other game theoretic concepts (Shapley-Value, Nucleolus)

- Assumption that the bargaining power can be measured by the positive and negative network effects

- requires data on coalitions

Thank you for your time and attention!

Malte L. Peters
Institute for Production und Industrial Information Management
University of Duisburg-Essen, Campus Essen
Universitätsstr. 9
45141 Essen, Germany
+49-(0)201-183 4167
malte.peters@pim.uni-due.de